Introduction (english translation)

In recent decades, there has been a growing research interest in "Mathematical Learning Disabilities" (henceforth noted MLD). Having this disorder implies having severe and/or persistent difficulties in learning mathematics (Deruaz et al., 2020). According to different studies, it affects between 5% and 8% of pupils (Butterworth, 2005; Geary, 2011; Lewis and Fisher, 2016; Noël and Karagiannakis, 2020) and is persistent throughout the schooling process (Geary et al., 2012), thus contributing to the creation of new educational and social inequalities.

At an institutional level, we are witnessing an intensification of the consideration of students with MLD [3] by political and educational authorities in different countries. For example, in Switzerland, in the canton of Vaud, the *Concept 360°* (2019) lays out solutions which appropriately address the diversity of pupils in an inclusive context. In France, the *Loi pour l'égalité des droits et des chances, la participation et la citoyenneté des personnes handicapées* (2005) and the *Loi pour la refondation de l'école* (2013) put forward an inclusive school adapting to the specific needs of each pupil. Or again, in Italy, Law 170/2010 (2010) regulates the educational measures, both compensating and dispensing, offered by schools to pupils with learning disabilities, and defines the possibilities for training the teaching staff on the subject, while taking into consideration the variety of actors: medical staff, school staff, pupils and families.

The concept of MLD is at the crossroads of several disciplinary fields. It is rooted in cognitive sciences as far as its definition (American Psychiatric Association, 2013; Organisation mondiale de la Santé, 2022) and diagnosis (Desmet and Mussolin, 2012) are concerned, but it is also closely linked to the field of education (Baccaglini–Frank, Di Martino, et al., 2020; Gardes and Prado, 2016; Giroux, 2011), thus impacting school reforms. Most of the current studies on MLDs are essentially situated in the field of cognitive sciences: neuropsychology and cognitive psychol-

3. We will refer to students using person-first formulations (https://apastyle.apa.org/style-grammar-guidelines/bias-free-language/disability) such as "students with MLD", which we considered more appropriate to the group of people studied for our work. We use the word "MLD" to indicate the disorder.

ogy (Giroux, 2011). Indeed, historically, the first traces of the notion of MLD date back to 1919, when the neurologist Henschen introduced the term "*akalkulia*" (Fischer, 2009b). Cognitive sciences are concerned with cognitive functioning and the characteristics of the individual. They link difficulties in mathematics with a dysfunction on the neurological level (Butterworth, 2005; Dehaene, 2010; Geary and Hoard, 2005).

Mathematics education takes up these questions, while bringing a point of view complementary to the cognitivist approach (Baccaglini–Frank, Di Martino, et al., 2020; Gauvrit, 2012). Indeed, in studying teaching and learning situations, mathematics educators consider the MLDs in terms of mathematical knowledge rather than in relation to the personal characteristics of the student (Giroux, 2011).

For example, Roiné and Barallobres (2018) take as their object of study special education classes in France and in Quebec by analysing the profile of the pupils attending these schools and the type of teaching they receive. The authors highlight certain phenomena that characterise special education, such as the impoverishment of the school knowledge to be transmitted, which can produce a cumulative effect leading to an aggravation of the pupils' difficulties.

Moreover, the vocation of mathematics education is to understand how to intervene in the face of pupils' difficulties (Baccaglini–Frank, Di Martino, et al., 2020). This intention is clear in the work of Robotti (2017) in Italy who asks, after identifying the main difficulties for students with MLD, "How can we didactically intervene in effective ways?" (p. 100). The researcher proposes a software to construct the meaning of some mathematical objects. The understanding of this meaning is enhanced by the variety of semiotic representations of the tool and the link that it allows between the student's sensory-motor system and the mathematical content.

Some research on MLDs is carried out in the field of mathematics education, at the interface with cognitive studies. For example, in her thesis, Peteers (2018) conducts a didactic analysis of diagnostic tests for MLDs, which are, for the most part, based on research in cognitive sciences. Furthermore, still by way of example, a Greek-Italian team (Karagiannakis et al., 2016) has designed a test for the identification of different MLD profiles with an approach at the crossroads between mathematics education and psychology.

Notwithstanding these few examples, research in mathematics education on MLDs deserves to be further developed. This thesis makes a contribution in this respect.

Indeed, in this manuscript, we are interested in the algebraic thinking of students with MLD. The choice of the mathematical focus is related to its importance

in schooling and to the lack of research in this area for pupils with MLD (Lewis and Fisher, 2016). In fact, almost all studies on MLD concern arithmetic in primary school. Yet, the difficulties of pupils with MLD are varied (Fias et al., 2013) and affect several aspects of mathematical skills (Kaufmann et al., 2013). In order to provide a didactic perspective and to broaden the scope of the analysed mathematical domains, we question, in our thesis, the competences and difficulties of pupils with MLD with regard to algebraic thinking.

To this end, in the first part of our work, after presenting the context and our research problem (chapter 1), we carry out two literature reviews. The first literature review concerns the MLDs and is situated at the interface between cognitive sciences and mathematics education, by articulating the two points of view (chapter 2). The second one, specific to mathematics education, concerns algebraic thinking, and leads us to study more specifically four aspects of it: generalisation; structure; argumentation; and the relation between quantities, with a focus on the relational interpretation of the equal sign (chapter 3).

With the aim of studying these aspects in students with MLD, and after presenting our theoretical supports (chapter 4), we pose a first research question (chapter 5):

Q1 What tasks should be created to analyse students' difficulties and skills specific to algebraic thinking?

To answer this, we design a battery of tasks using Kaput's model of algebra (Kaput, 1995, 2008), the core of which is generalization. The eight tasks chosen are described and analysed in the second part of the thesis (chapters 6 and 7). They are proposed during clinical interviews to eighteen secondary school students: fifteen with MLD and three without MLD. The experimentation is presented in the third part (chapter 8).

We analyse the interview transcripts using three frameworks: Malara and colleagues' language constructs (Cusi et al., 2011; Malara and Navarra, 2018), Radford's generalisation types (2001, 2008) and Balacheff's types of proofs (Balacheff, 1987). This analysis allows us to answer the following research question and its subquestions:

Q2 How can we describe the algebraic thinking of students with MLD in terms of language constructs, types of generalisation and types of proof?

Q2.1 Which language constructs identify the algebraic thinking of students with MLD?

Q2.2 What types of generalisation are used by students with MLD in the study of a geometric pattern?

Q2.3 How do students with MLD mobilise examples and engage in proofs in an early algebra task requiring argumentation?

The analyses of the transcripts of the interviews and of the written productions highlight the ability of pupils with MLD to take an algebraic look at the tasks proposed by resorting to a relational vision of the equal sign and by focusing on the process and the structure of a certain task (chapter 9). Furthermore, they show that students with MLD implement several types of generalisation (arithmetic, factual algebraic, contextual algebraic, etc.) and different functions of the examples.

In conclusion, the results indicate that students with MLD can use algebraic thinking. This does not differ qualitatively from that of students without MLD in terms of procedures and difficulties. What seems to distinguish the two categories is rather the frequency and persistence of difficulties even following the inputs of the researcher during the experiment. We detail these results in the third part.

These results open the way to the design of didactical proposals aimed at the development of algebraic thinking and intended for classes with students with MLD. We will discuss this in the fourth part of the thesis, where we also examine the scope and limitations of our work (chapters 10, 11 and 12).

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