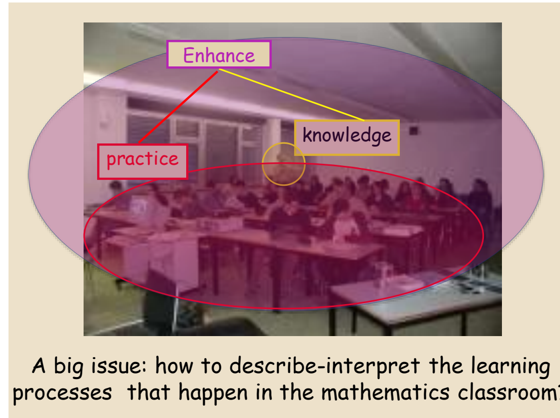
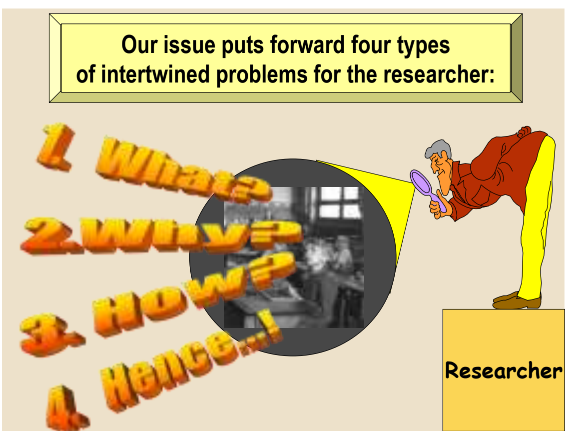
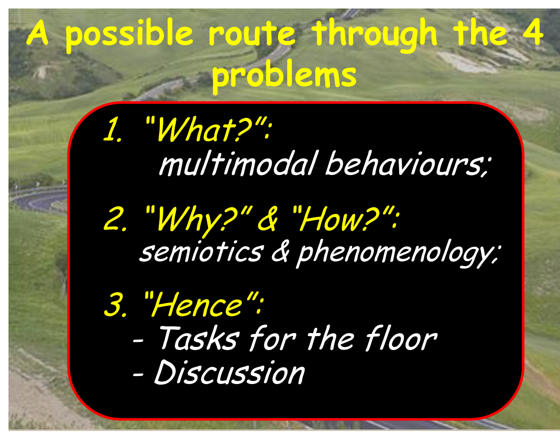
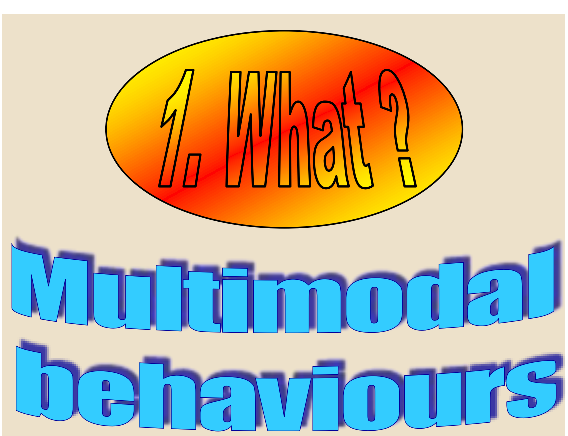
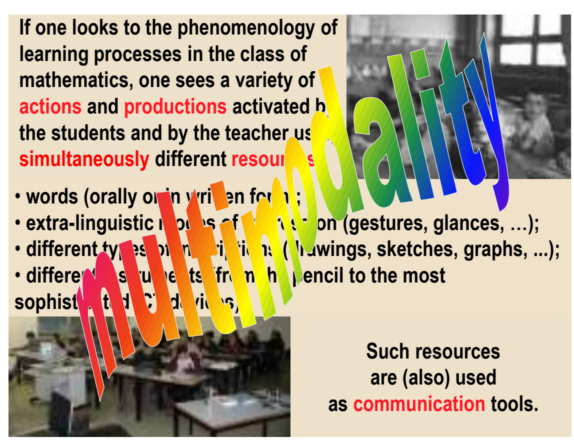

Ferdinando Arzarello
Dipt. di Matematica
Università di Torino
In ricordo di G. Prodi, il Maestro di tutti noi, perché la nostra comunità possa crescere con lo stile di serietà e sobrietà che Giovanni le diede.
26 giugno 2015
Pisa
Scuola estiva in didattica della Matematica

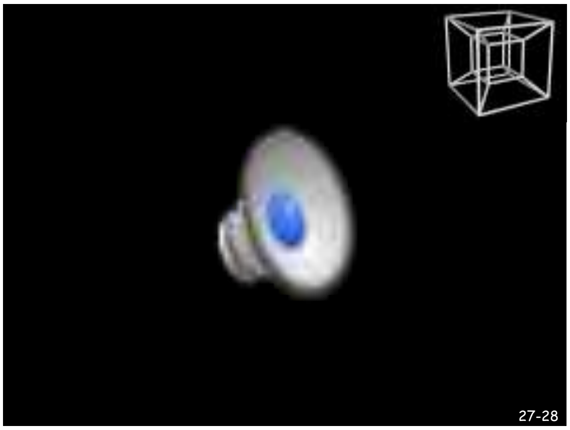

Enhance
knowledge
practice
A big issue: how to describe-interpret the learning processes that happen in the mathematics classroom?


Our issue puts forward four types of intertwined problems for the researcher:
1. What?
2. Why?
3. How?
4. Hence...!
Researcher


A possible route through the 4 problems
1. "What?": multimodal behaviours;
2. "Why?" & "How?": semiotics & phenomenology;
3. "Hence": - Tasks for the floor - Discussion


1. What ?
Multimodal behaviours


If one looks to the phenomenology of learning processes in the class of mathematics, one sees a variety of actions and productions activated b
the students and by the teacher us
simultaneously different resour
multimodality
• words (orally o
• extra-linguistic
on (gestures, glances, ...);
• different ty
wings, sketches, graphs, ...);
• differe
encil to the most
sophist
Such resources are (also) used as communication tools.



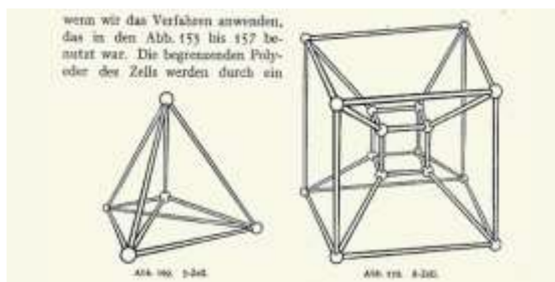
DIE GRUNDLEHREN DER
MATHEMATISCHEN
WISSENSCHAFTEN

W. BLASCHKE M. BORN

R. COURANT

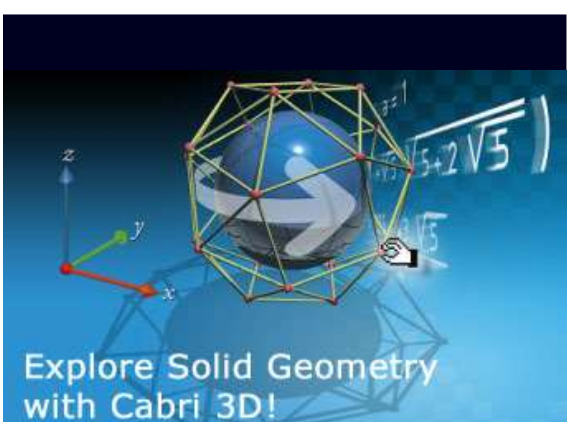
ANSCHAULICHE GEOMETRIE

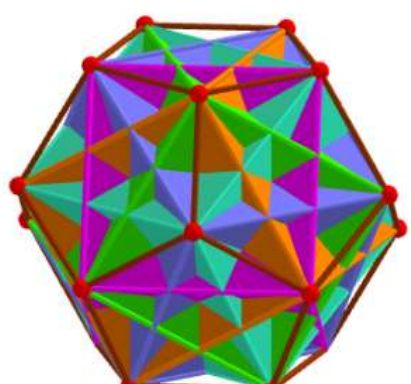
BERLIN
VERLAG VON JULIUS SPRINGER



In Cabri after various attempts we decided to represent a plane as a rectangle (in some earlier version as a parallelogram) presenting a certain amount of thickness, in the very same spirit as when Hilbert and Cohn-Vossen designed their 3D figures for their famous “Anschauliche Geometrie” (Hilbert & Cohn Vossen, 1952).

(J.M. Laborde)





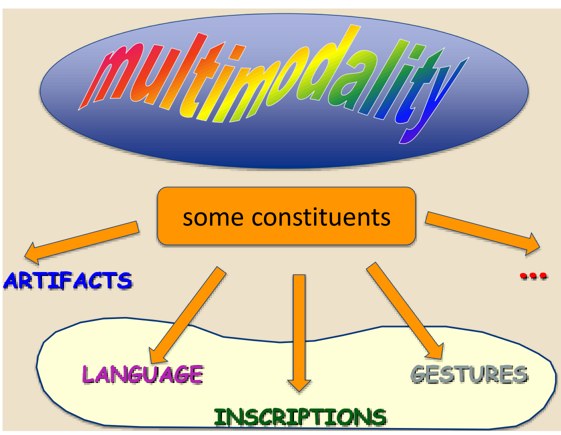
Mathematical meanings that arise in teaching and learning are of a multimodal nature.

→ new understandings of the role of the body, language, and material culture.

These conceptions highlight the cognitive role of semiotics and embodiment in mathematics thinking, teaching, and learning.

Within these new conceptions, gestures, body posture, kinesthetic actions, artifacts and signs in general are considered as a fruitful array of resources to be taken into account in order to investigate how students learn and how teachers teach.

These sensible and material resources are not considered as a mere epiphenomenon of teaching and learning: they are conceptualized as central elements of the students' and teachers' mathematical thinking.



Gesti

DEFINIZIONE
Movimenti di mani e braccia mentre si parla.

I gesti sono collegati al parlato dalla definizione stessa

L'interesse per i gesti nasce in ambito psicologico, in relazione alle diverse ipotesi sull'influenza del linguaggio sul pensiero.

Caratteristiche dei gesti

Dinamica:
- Partono da e ritornano a una "posizione di riposo"
- Hanno una struttura "ad apice", una fase centrale saliente ("stroke phase")
- Sono bene delimitati (chiare fasi di inizio e fine)
- Hanno una struttura simmetrica
- La stroke phase è sincronizzata il contributo linguistico più significativo

Il "gesture space"

La maggior parte dei gesti avviene in uno spazio delimitato, che McNeill chiama "**gesture space**":
Tra le spalle e dagli occhi alla vita.

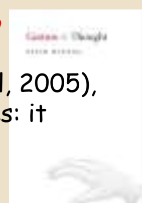
PENSARE

GESTI

LINGUAGGIO

- Come sono coinvolti i gesti nel processo di produzione linguistica?
- Più in generale, qual è la funzione cognitiva dei gesti?

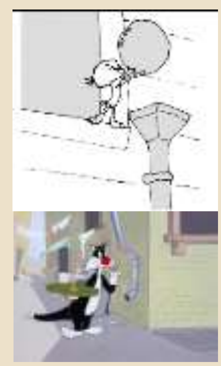
L'unità gesto-linguaggio



Gesture and Thought (D. McNeill, 2005), is a milestone for gesture studies: it establishes a new conception of language: language as an

imagery-language dialectic.

In this dialectic gestures provide the imagery, and the dialectic itself fuels speech and thought: gesture is a 'material carrier' of imagery (Vygotsky 1986).



"and he goes up thròugh the pipe this time"

In speech, meanings are analyzed and segregated.
Speech divides the event into semantic units—a directed path ("up"), plus the idea of interiority ("through").
Analytic segregation further requires that direction and interiority be combined, to obtain the composite meaning of the whole.

In gesture, this composite meaning is fused into one symbol and the semantic units are simultaneous—there is no combination (meaning determination moves from the whole to the parts, not from the parts to the whole).

The effect is a uniquely gestural way of packaging meaning—something like 'rising hollowness', which does not exist as a semantic package in the English lexicon at all.

Thus, speech and gesture, at the moment of their synchronization, were co-expressive but non redundant, and this sets the stage for doing one thing (conception of the cat's climbing up inside the pipe) in two forms—analytic/combinatoric and global/synthetic.

Gesture is an integral component of language in this conception, not merely an accompaniment or ornament.

Such gestures are synchronous and co-expressive with speech, not redundant, and not signs, salutes, or emblems.

They are frequent—about 90% of spoken utterances in descriptive discourse are accompanied by them.

They occur in similar form across many cultures. They synchronize with speech at points where speech and gesture embody shared underlying meanings in discourse, and possess communicative dynamism, points of maximal discursive force at the moment of speaking.

A host of phenomena reveal the tight bond—to the point of fusion— of speech-gesture combination at these points. This fusion creates conditions for a dialectic. The dialectic is of unlike cognitive modes.
Speech and gesture contrast semiotically:
- a gesture is holistic (i.e. global and synthetic), instantaneous, and not specified by conventions of form;
- a linguistic form is analytic, combinatoric, linear, and defined by socially-constituted rules.

McNeill's classification

- **Iconic**: the imagery underlying the motion depicts form or manner of aspects of the accompaying speech ("representational gestures")
- **Metaphoric**: the same but for abstract aspects
- **Deictic**: to indicate objects, events, or locations in the concrete world
- **Cohesive**: tie togheter temporally separated part of discourse
- **Beat**: motions to stress segment of discourse, speech rythm

Due elementi dinamici

1. Growth point

it is the initial form of a dynamic unit of online verbal thinking out of which a dynamic process of organization emerges.The GP combines unlike modes of cognition imagery and linguistic categorial content.
Growth points are considered to be psychological predicates and are, accordingly, units of inner speech, in Vygotsky's terms.
However, unlike inner speech, they are not spoken or even necessarily speakable—to get to speech, something further takes place, and this is what will be termed *unpacking*.

Consistent with a dialectical approach to thinking communication, McNeill thought of the growth point as something like a seed with a communicative potential that concretely realizes itself in subsequent sign production.

Thus, the original form in which an idea first appears at the growth point and its reiterated forms in the catchments stand in a genetic family relationship that underlies the dialectical method (Il'enkov 1977).

Due elementi dinamici

2. Catchment

A catchment is recognized when one or more gesture features occur in at least two (not necessarily consecutive) gestures. The logic is that recurrent images suggest a common discourse theme, and a discourse theme will produce gestures with recurring features. These gesture features can be detected. Then, working backwards, the recurring features offer clues to the cohesive linkages in the text with which they co-occur. A catchment is a kind of thread of visuospatial imagery that runs through a discourse to reveal the larger discourse units that encompass the otherwise separate parts.

Growth Points and Catchments in the classroom

In the study of classroom communication, one can observe a complex organization of a large range of signs that appear to contribute to the meaning-making efforts of teachers lecturing to students and to students' productions while communicating each other or with the teacher. This organization is evident to the analyst, e.g. through the identification of recurrent themes in the sign production of the teacher (or of the students), which carries forward one idea in his (their) discourse until it possibly achieves stabilization.

Recent research has shown that gestures, inasmuch an important component of cognition, are important also in science and mathematics learning.

(McNeill, 1992, 2005; Lakoff & Nunez, 2000; Goldin-Meadow, 2003; Arzarello et al., 2009)

People gesticulates also while doing mathematics

From Science Education

"…gestures express new levels of understanding before a student expresses this new understanding in words; more so, gestures express the new concepts although language still holds on to the old, incorrect concepts."

"… students are attuned to the gestures teachers use and sometimes appropriate these gestures into their own expressive repertoires, thereby accelerating the development of scientific literacy."

W.-M. Roth, *Making Use of Gestures, the Leading Edge in Literacy Development* (2003)

From Mathematics Education

"Thinking… does not occur solely in the head but in and through language, body and tools. As a result and from this perspectives, gestures, as a type of bodily action, are not considered as a kind of window that illuminates the events occurring in a "black box" – they are not clues for interpreting mental states. They are rather genuine constituents of thinking."

(Radford, 2009, p. 111)

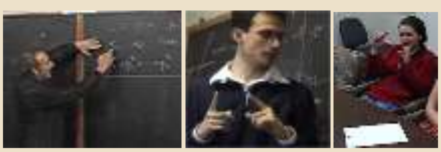
Our main questions:

- Do exist specificities of gesturing in mathematics, which distinguish them from everyday gesticulation?
- How do gestures enter into students' mathematical conceptualization processes?
- How such an analysis can help math educators in their research and teachers in their job?

Edwards, Radford & Arzarello, ESM special issue, 2009, 70(2).

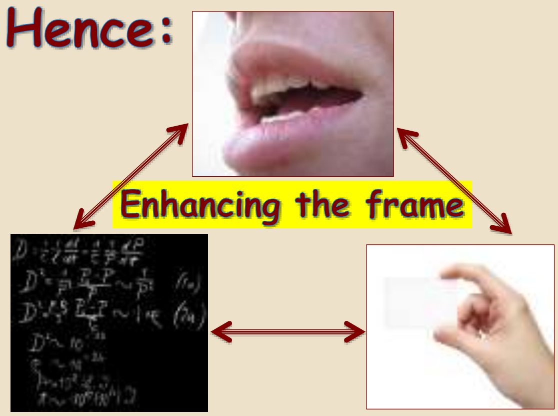
Our main result:

Gestures in mathematics are deeply intermingled not only with speech but also with the inscriptions produced/acted on by people while solving problems.



Edwards, Radford & Arzarello, ESM special issue, 2009, 70(2).

Hence:



multimodality

An example
(from a joint research)

As a preliminary study in a research aimed at studying the role of graphical and visual components for grasping elementary calculus concepts, two NZ secondary school mathematics teachers, Ava and Noa, have been asked to solve some contextualized "graphical" problems (no formulae but only graphs of functions), whose mathematical content was:

Given (the graph) f draw f' (derivative fcn.)
Given f draw F (an antiderivative of f)

Ava and Noa:

- Taught mathematics up to year 12, but had not taught year 13 calculus (final year of secondary school in New Zealand).
- Knew well each other.
- Ava had 7 years of teaching experience, Noa had 2 years of teaching experience.
- Ava and Noa worked together on the tasks over the course of an hour and were video-taped and audio-taped.

Ava

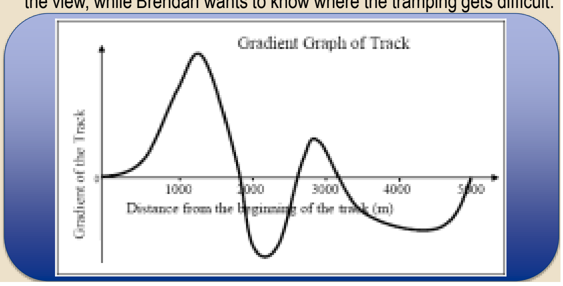
Noa

The main task

Problem

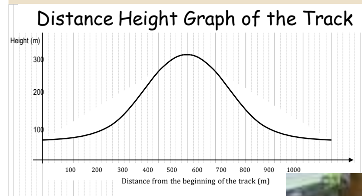
To celebrate their 40th wedding anniversary, Helen and Brendan O'Neill are planning a tramp with their children and grandchildren.

The local park provided a Gradient Graph for a nearby 5-kilometre tramp, but the O'Neills want to make sure it is suitable for them. Helen wants to know if there is a summit where they can have lunch and enjoy the view, while Brendan wants to know where the tramping gets difficult.



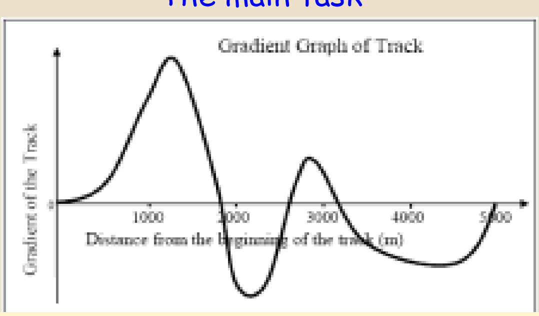
Previously A & N had solved a "Warming up" question:

Distance Height Graph of the Track



Find the Gradient-Height graph

The main task



I will show you what happened after a while (≈3:30-5:45) with a video-clip.

Videoclip 1

Comments from the floor, please!

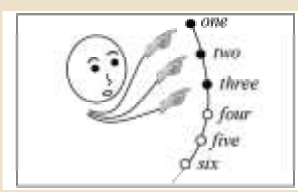
The virtual space

Virtual space:

Mathematical space
Cognitively maintained
Constrained by physical gesture space

A virtual space is created through gestures

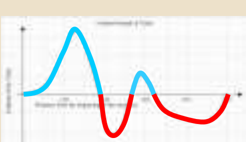
A virtual space becomes a framework for interpreting gestures

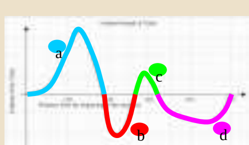


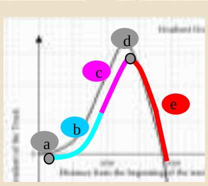
1. A primitive virtual space

Noa: But just generally on a graph… plus gradient is up, and negative gradient is down.

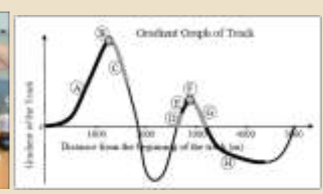
2. Verbal descriptions of the tramping track, deictic gestures







3. Creating the virtual space



Noa: Steep hill [*traces along A and towards B*] and still going up [*traces along C*] but not as sharply.

3. Creating the virtual space

4. Recreating the virtual space

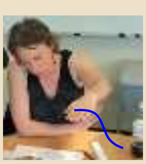
5. Deictic gestures within the virtual space

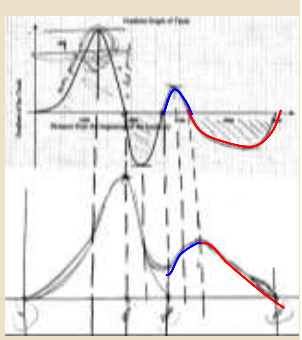
Noa And then we start going back up, gentle to a harder point.
Ava But not as hard as it was over there [points in the air to the other steep slope Noa had acted out].
Noa But not as hard as it was over there.

5. Deictic gestures within the virtual space

Noa: And then gradient is getting easier. So, flattening off [*Ava points in the air to where Noa's hand is gesturing so as to extend the imaginary slope further*]. We're still going up but we're flattening off …

6. Using the virtual space for difficult tasks

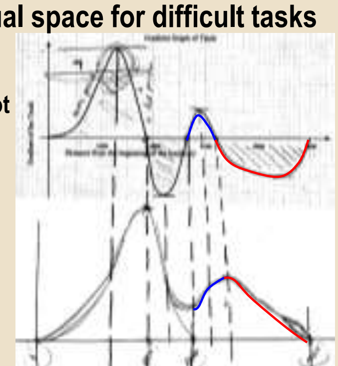




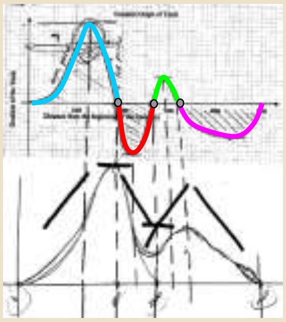
Noa: So you're going up not so steeply, becoming steeply, point of inflection. Not so steeply, so not so steeply, steeper and then going up again

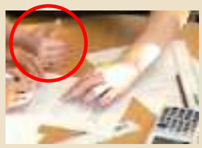
6. Using the virtual space for difficult tasks

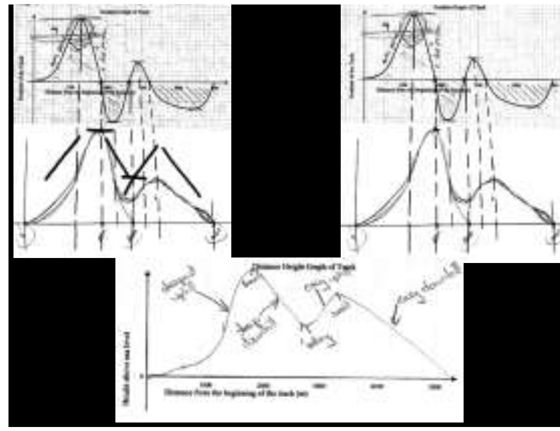
Noa: And it's all downhill, downhill, not so steeply, constant steepness, not so steeply, so, "neeauuuurrr"



7. A horizontally oriented virtual space

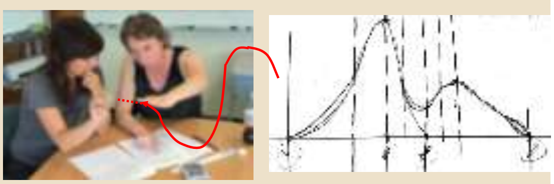






What are some implications for practice?

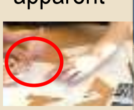
Learning potential:



The experimental nature of a temporary space

ALCUNI COMMENTI GENERALI

The multimodal use of different resources in A&N solution process is apparent



Moreover, while in the first part A & N use mainly deictic/iconic gestures on the graph, in the part shown at the video the role of gestures has become more complex: the imagery-language dialectic assumes a "mathematical" specificity within the virtual space.

We need a unitary analysis tool suitable for focusing:
- the phenomenology of such processes;
- both the holistic (e.g. of gestures, sketches) and the recursive (e.g. of natural, mathematical languages) nature of its components;
- the mutual relationships of their constituents.

Focusing learning processes with a multimodal lens will have important theoretical and pragmatic consequences on the way we:

- conceive the mathematical objects to teach;
- collect our data;
- plan our researches and design the teaching sessions in the classrooms.

What?

Why? & How?

Hence!

A possible route through the 4 problems

1. "What?": multimodal behaviours;
2. "Why?" & "How?": semiotics & phenomenology;
3. "Hence": - Tasks for the floor - Discussion

2. Why and How?

R. Duval: "there is no noesis without semiosis"

"Beyond the traditional psychological focus on mental structures and functions it [semiotics] considers the personal appropriation of signs and the underlying meaning structures embodying relationships between signs. …
a semiotic approach draws together the **individual** and

A semiotic approach "includes **cultural** significations such as the concepts surrounding mathematical thought (its naturalness, its way of existing, its relation to the concrete world, etc.) and social patterns of the production of meaning."

Radford (2006)

(see ESM, 2006; RELIME, 2006)

Semiotic systems provide an environment for facing mathematics not only in its structure as a scientific discipline but also from the point of view of its learning, since they allow us to seek the cognitive functioning underlying the diversity of mathematical processes.
In fact, approaching mathematical activities and products as semiotic systems also allows us to consider the cognitive and social issues which concern didactical phenomena

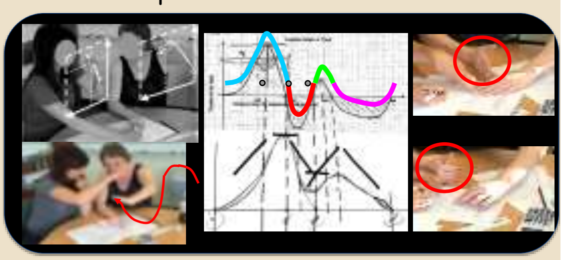
But to do that, a broader notion of semiotic system is necessary.

In fact the classical semiotic approaches place strong limitations upon the structure of the semiotic systems they consider: e.g. see the description given by Duval or Ernest (ESM, 2006).

E.g. according to Ernest (2006, pp. 69-70), a *semiotic system* consists of three *components*:

- A set of ***signs***, the tokens of which might possibly be uttered, spoken, written, drawn or encoded electronically.
- A set of ***rules*** of sign production and transformation, including the potential capacity for creativity in producing both atomic (single) and molecular (compound) signs.
- A set of ***relationships*** between the signs and their meanings embodied in an underlying meaning structure.

The classical semiotic systems result too narrow for interpreting the complexity of didactical phenomena in the classroom.



The classical semiotic lens is blind with respect to a lot of semiotic resources that are active in the classroom.

But we have seen the relevance of

For developing a suitable analysis, a broader notion of semiotic system is necessary!

such non verbal semiotic resources as gestures, gazes, sketches…in learning processes.

The Semiotic Bundle is this broader integrated analsis tool

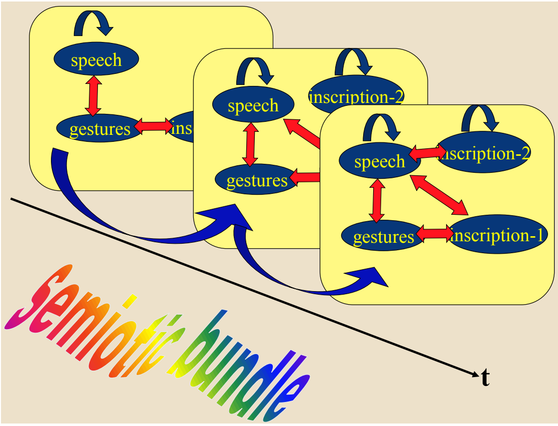
It enlarges the classical notion of semiotic system within a Vygotskyan perspective.

Arzarello (2006). Semiosis as a Multimodal Process. RELIME, Num. Especial, p. 267-299

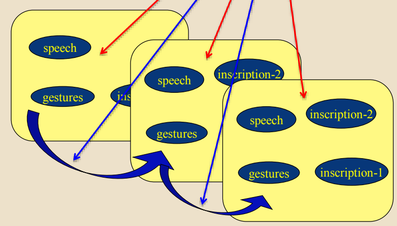
The Semiotic Bundle

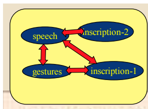
It reveals particularly suitable

- for grasping the **multimodality** of the learning processes and the different forms of **attention**
- for setting them according to the multimodal paradigm.

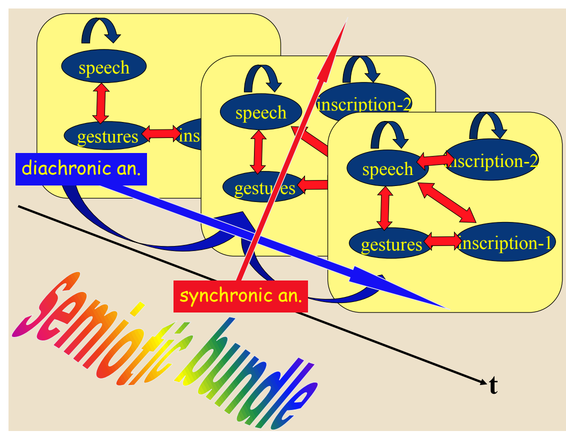


A semiotic bundle is:
(i) A collection of semiotic sets;
(ii) A set of relationships between the sets of the bundle
(conversions/treatments)



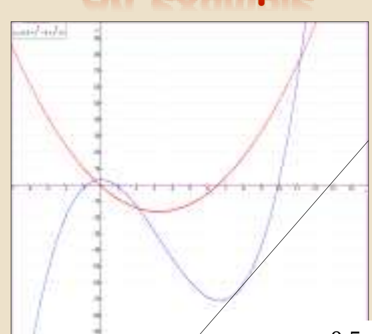


A semiotic set is:
a) A set of signs which may possibly be produced with different actions that have an intentional character, such as uttering, speaking, writing, drawing, gesticulating, handling an artefact.
b) A set of modes for producing signs and possibly transforming them; such modes can possibly be rules or algorithms but can also be more flexible action or production modes used by the subject
c) A set of relationships among these signs and their meanings embodied in an underlying meaning structure.



Analysing the dynamics of the different semiotic resources used by the students through the Semiotic Bundle allows us to enter into their processes of solution and to focus the richness and complexity of the multimodal shared cognitive environment, where their resources develop.

An example



$y = 0{,}5\ x^3 - 5x^2 + 3\backslash$

PART 1

1. Ciro: Practically one could say to explain that this straight line…
2. Teacher: Uh
3. C: That is it must join, ok, the X axis the interval...is [always?] the same **A**
4. Teach.: The X interval is the same; delta-X [Δx] is fixed
5. C: Delta…eh, indeed, however there are some points where… to explain it one can say that this straight line must join two points on the Y axis, which are farther each other, hence it is steeper towards...
6. Teach.: Yes
7. C: Let us say towards this side. When, here, …when …however it must join two points, which are farther, hence there is less distance **B**
8. Teach.: More distance?
9. C: Less far [he corrects what said in #7]
10. Teach.: Eh
11. C: On the Y axis I am saying
12. Teach.: Yes
13. C: It slants softly from this side; in fact here is the point, let us say...we may call it zero, since this is a real line like so.

THE BASIC SIGN (A)

The inscriptions are extracted from the screen and embodied in Ciro's gesture.

The evolution of the gesture from Fig. 1 to Fig. 4 illustrates a concept not present in the words: the line is joining two points whose *x*-coordinates are very close, but it is not the same for the corresponding *y*-coordinates. ***That is it must join, ok, the X axis the interval...is [always?] the same***

Ciro's words (see A) refer only to the (quasi) tangent line and express the fact that the interval is always the same. For this reason, we call the gesture in Fig. 1 the *basic sign*: in fact it triggers a semiotic genesis of signs, which will span over all the other episodes of this activity and beyond.

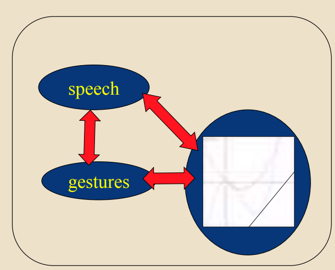
The semiotic bundle is made of:

- the inscriptions on the screen (the two dynamic graphs and the moving tangent line);
- Ciro's gestures and words;
- the teacher's words.

The analysis of the bundle points out some different ingredients introduced by Ciro to explain how the tangent is varying: namely the fact that for equal and small Δx's the corresponding Δy is different (e.g. bigger and bigger when the slope increases). Gesture and speech convey different information (it is a case of non-redundant gesture): this information will be explicated verbally in the following Episode B.

THE COVARIATION BETWEEN Δx and Δy (B)

Some relationships between the resources are shown by a synchronic analysis.



Ciro's attention is concentrated on the relationships between the Δx and the corresponding Δy variations.

Gesture and speech convey the same information: they express the covariation between Δx and Δy, underlining the case when bigger variations of Δy correspond to small values of Δx.

This description involves gestures and words, simultaneously used in the activity.

Ciro's actions have a multimodal feature: the student is speaking and simultaneously gesturing conveying the same meaning in two ways. But gestures (and also words) run an evolution from Episode A to Episode B, which can be described by a diachronic analysis.

The gestures in Episode A have a genetic role with respect to the words in Episode B: the latter explicates the former within a different semiotic register.

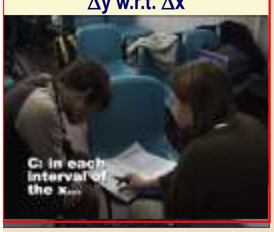
Diachronic analysis in the short or in the long period shows other relationships between the resources.

Long period:

From the past the culture of the classroom (finite differences) appears.

It is expressed through words and gestures

COMPARING DIFFERENCES: Δy w.r.t. Δx



Using the Semiotic Bundle lens it is possible to focus important didactic phenomena.

Using the Semiotic Bundle lens it is possible to focus important didactic phenomena.

I will show an example: the semiotic game

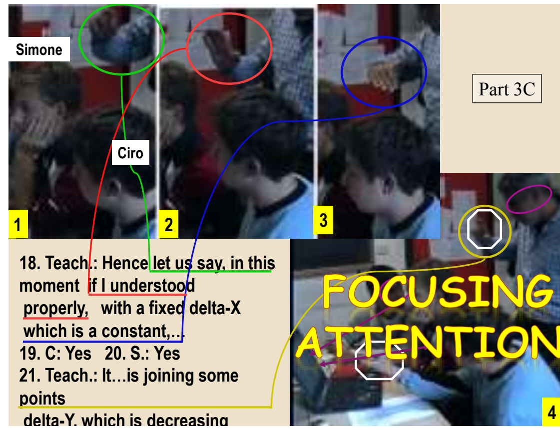
PART 3

18. Teach.: Hence let us say, in this moment if I understood properly, with a fixed delta-X, which is a constant,…
19. C: Yes
20. S: Yes
21. Teach.: It… is joining some points with delta-Y, which are near

C

22. [overlapped with #21] C: In fact, while they [the points on the graph] are approaching each other
23. C:…they [their ordinates] are less and less far. In fact, I do not know how to say it,…the slope is going towards zero degrees.
24. Teach.: Uh, uh
25. C: Let us say so
26. Simone: Ok, at a certain point here delta-Y over delta-X reaches…
27. C: The points are less and less far
28. Teach.: Sure
29. S:…a point, which is zero.

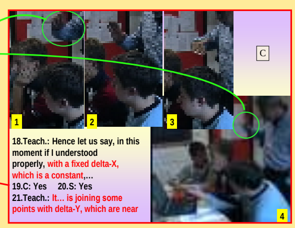
D



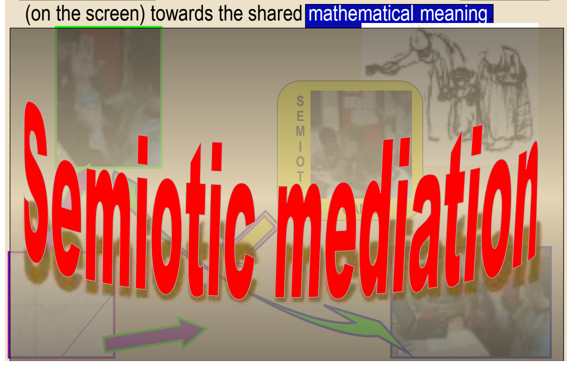
The semiotic game

The teacher uses:

1. the gesture register to be in tune with his students: he "echoes" their gestures;
2. the verbal register to express the rigorous mathematical meaning, different from that used by the students.



Through the semiotic game the teacher pushes the personal senses that the students have attached to the initial sign (on the screen) towards the shared mathematical meaning



The episode shows that **communication** is particularly active in this moment.

We have entered into it through the semiotic lenses, but the real sense of what is happening is given considering the interaction between the teacher and the students and the way the teacher is coaching it, within his didactic strategy and design.

The effects of the semiotic game

S. uses the words previously introduced by the teacher (# 18, 21) and converts what Ciro was expressing in a multimodal way through gestures and (metaphoric) speech into a fresh semiotic system. His words in fact are an oral form of the symbolic language of mathematics: the semiotic system is not any longer the everyday language but already the language of Calculus.

C is looking carefully to what S is doing and saying, comments *"The points are less and less far"* and repeats again a gesture, which is a small variation of the basic one. Namely he intertwines the representations within his semiotic bundle with those used by S.

Using the Semiotic Bundle lens it is possible to focus important didactic phenomena.

For example the semiotic game driven by the teacher

Personal Vs/ institutional signs:

Sometimes the personal signs of the students (e.g. gestures and/or personal inscriptions) are able to support their transition to the institutional ones and to the sharing of common treatment rules.

Other times this does not happen and the mediation of the teacher becomes necessary, e.g. through a **semiotic game**.

Personal Vs/ institutional signs:

Teaching-learning consists in students' personal appropriation of the sign's meaning, fostered by social interactions, under the coaching of the teacher.

As a consequence, their gestures within the Semiotic Bundle become a powerful mediating tool between signs and thought, which supports the evolution towards the institutional signs.

Personal Vs/ institutional signs:

From a functional point of view, gestures can act as "personal signs", while the semiotic game of the teacher can start from students' gestures and use the right words to support their transition to the scientific meaning.

Semiotic games constitute an important strategy of the teacher for triggering students' appropriation of the culturally shared meaning of signs.

They allow the teacher to become suitably in tune with students' languages, and conversely they allow students to achieve resonance with the teacher's languages and, through them, with the institutional knowledge.

A phenomenological perspective

Say/Explain What you **See**
(how it makes sense for you):

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A phenomenological perspective

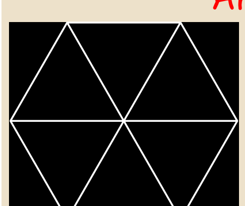
Say/Explain What you **See**
(how it makes sense for you):

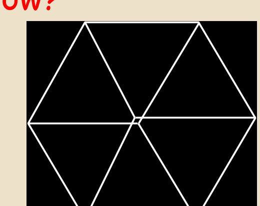
A phenomenological perspective

What **is** it?

Say/Explain What you **See**
(how it makes sense for you):

And now?





A phenomenological perspective

Prompting (focusing) the students' attention to the suitable context, possibly enriched with recalled or imagined elements, supports the students towards a progressive ***disclosure*** of the mathematical objects at stake.

- Radford, L. (2010). The eye as a theoretician: seeing structures in generalizing activities. *The learning of Mathematics*, (Vol. 30 pp. 2-7). Edmonton: FLM Publishing Association.
- Mason, J. (2008). Being Mathematical With & In Front of Learners: Attention, Awareness, and Attitude as sources of Differences between Teacher Educators, Teachers & Learners. In T. Wood (Series Ed.) & B. Jaworski (Vol. Ed.), *International handbook of mathematics teacher education: Vol.4.* (pp. 31-56.). Rotterdam, NL: Sense

Esplicitate il senso di queste formule:

$$\frac{\sqrt{3}+\sqrt{2}}{(\sqrt{3}-\sqrt{2})(5+2\sqrt{6})} = 1$$

$$\sum_{k=\sum_{1}^{n-1} i}^{\sum_{1}^{n-1}(i+1)} (2k+1) = n^3$$

Mason, Int.l Handbook… 2008, 31-56; Arcavi, 1994, FLM, 24-35

$$\frac{\sqrt{3}+\sqrt{2}}{(\sqrt{3}-\sqrt{2})(5+2\sqrt{6})} = 1$$

$$\sum_{k=\sum_{1}^{n-1} i}^{\sum_{1}^{n-1}(i+1)} (2k+1) = n^3$$

Gazing at the Whole?

Discerning Details?

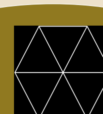
Recognising Relationships?

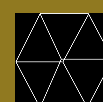
Perceiving Properties?

Reasoning on the basis of Properties?

Mason, Int.l Handbook… 2008, 31-56

Seeing something means…



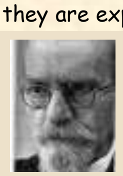


Disclosure

Mason, Int.l Handbook… 2008, 31-56; Arcavi, 1994, FLM, 24-35

A phenomenological perspective

Disclosure is a Husserlian concept further elaborated by Rota (1991) for mathematics. It indicates the process by which people **make sense** of the world and of the situations in context to which they are exposed.



Edmund Husserl
(1859 - 1938)

Giancarlo Rota
(1932 - 1999)

"Sense-making depends ultimately on our own being-in-the-world, on the situation of our interacting, our dealing with the contextual situation in the world […]. If you deconstruct the notion of an object, what you find is pure functionality, the pure 'being good for' of that object or something. So that the world, instead of being a world of objects, will become a world of functions, of tools".

Such functions are related to each other "*by a system of references, a network of references among them. […] The world is disclosed to us not just as a system of functions, but as a network of related functions*".

G.C. Rota, The end of objectivity, *Lectures at MIT*, 1974-1991 (pp. 156-159)

The understanding of the sub-formulas and formula in the exercise requires to perceive them in a certain way.

$$\sum_{k=\sum_{1}^{n-1} i}^{\sum_{1}^{n-1}(i+1)} (2k+1) = n^3$$

The understanding of the sub-formulas and formula in the exercise requires to perceive them in a certain way.

"To attend something in a certain way, and to "intuit" it in a specific form, to use Husserl's concepts, requires a specific intentional act. The object as it appears in consciousness and the intention to perceive it are coterminous. The problem was then to account for that which would allow the students to move from an initial perception [...] into a more sophisticated one."

Such a transformation is crucial to ensuring subjects' increased fluency in a complex form of mathematical reasoning vital to their road towards algebra.

"This transformation [...] is part of the *domestication of the eye*, or – if one so prefers – in educating the eye, that is to say, in converting it into a cultural-theoretical organ of perception."

"[...] the mathematicians' eyes have been culturally educated to organize the perception of things in particular *rational* ways.

The mathematicians' eyes have undergone a lengthy process of domestication. That such a process is not "natural" is proven not only by results from cross-cultural psychology (Geurts, 2002; Segall, Campbell, & Herskovits, 1966) but also by our young students' responses.

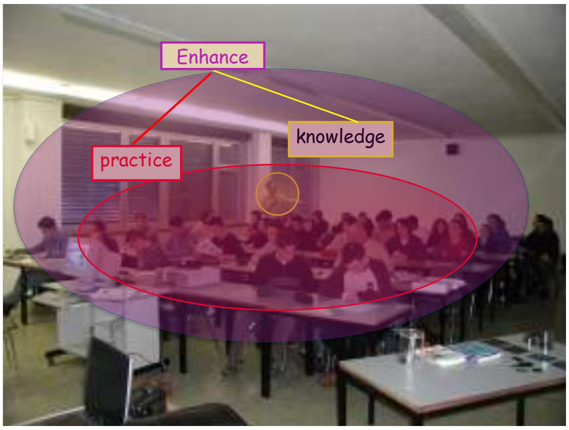
The domestication of the eye is a lengthy process in the course of which we come to see and recognize things according to "efficient" cultural means. It is the process that converts the eye (and other human senses) into a sophisticated intellectual organ – a "theoretician" as Marx put it (Marx, 1998).

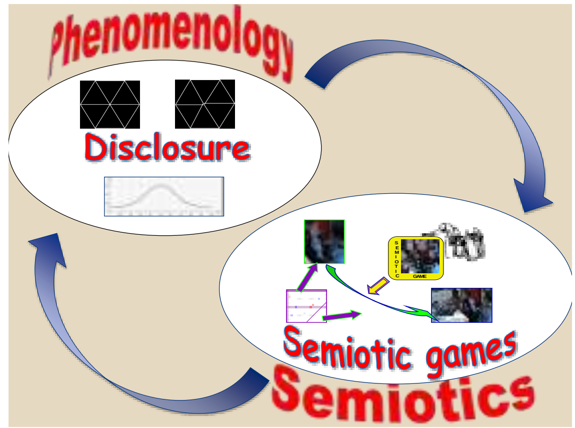
[..] The capacity to perceive certain things in certain ways, the capacity to intuit and attend to them in certain manners rather than others, belongs to those *sensibilities* that students develop as they engage in processes of objectification."

L. Radford. The eye as a theoretician: seeing structures in generalizing activities, FLM 30, 2 (July, 2010), 2-7.

The teacher's role: a semiotic mediation (e.g. through semiotic games) in order to promote the disclosure of math objects in the classroom.

Teachers can "*create the possibility for students to perceive things in certain ways and encounter a cultural mode of generalizing. This new way of perceiving (...) in certain efficient cultural ways entails a transformation of the eye into a sophisticated theoretician organ*".
(Radford, The eye…, p. 2).





A possible route through the 4 problems

1. *"What?": multimodal behaviours;*
2. *"Why?" & "How?": semiotics & phenomenology;*
3. *"Hence":*
   - *Tasks for the floor*
   - *Discussion*

Laboratorio

Nella lezione avete visto alcuni strumenti di analisi semiotica e fenomenologica, utili per progettare o interpretare i fenomeni didattici in classe:

- Semiotic bundle
- Catchment
- Growth point
- Disclosure
- Education of eye
- …

Ora vi si propone di usarli per interpretare alcuni episodi di vita di classe.
Fatelo lavorando in gruppo e poi discutiamo insieme i vantaggi e le carenze di analisi che trovate in questo lavoro interpretativo.

Compito 1

"La coperta di Penelope"

Penelope radunò subito i pretendenti e disse loro: "Ho deciso, sceglierò tra voi il mio sposo e le nozze si celebreranno quando avrò finito di tessere una nuova coperta per il letto matrimoniale. Incomincerò oggi e prometto di tessere ogni due giorni; quando avrò finito, la coperta sarà la mia dote di sposa".
I pretendenti accettarono. La coperta doveva essere lunga 15 spanne. Penelope iniziò subito il lavoro, ma un giorno tesseva una spanna di coperta, mentre il giorno dopo, di nascosto, ne disfaceva la metà….

Liberamente tratto dall'Odissea

Penelope dovrà sposare uno dei pretendenti, o Ulisse arriverà prima che la coperta sia completata?

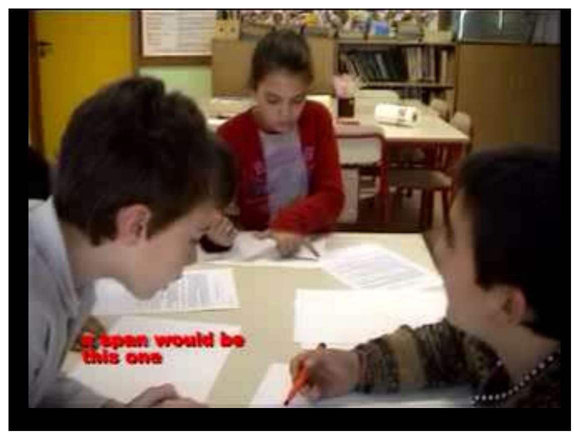
Videoclip 1, 2

Use the Semiotic Bundle Model to interpret the dynamics of what happens:

- Point out aspects of strength of the model
- Point out aspects of feebleness of the model

Videoclip 3

- Specific differences from Videoclips 1, 2: comments and hypotheses



Compito 2

Using suitable theoretical frames compare what happens in the two classrooms:

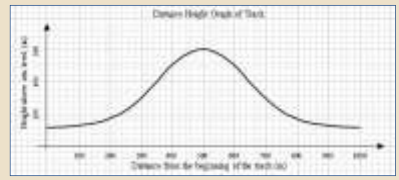
- What semiotic resources are used? How?
- Try a phenomenological analysis
- How to intertwine the two ?

Case study

The task (solved in group-work):

FIRST PART

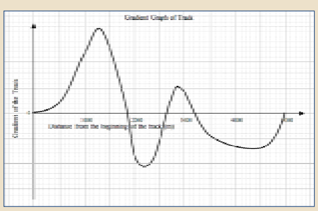
- To interpret a height-distance graph and to draw the graph that represents its slope (MODELLING PROBLEM)



Case study

SECOND PART:

- To draw a graph whose slope was represented by the following gradient graph (INVERSE PROBLEM)



Compito 3

La corsa a 20 (2^a elem.)

REGOLE DEL GIOCO:

- Due giocatori
- Ogni giocatore a turno aggiunge un numero tra 1 e 2 (non si possono saltare turni)
- Si parte da 0
- Il giocatore che arriva a 20 vince.

G. Brousseau (1998)

Strategia vincente

Si può giocare solo 1 o 2

G's ARGUMENTS

Video 1

che è un numero vincente, lui aggiunge 1 o 2, io

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E's ARGUMENTS

Video 2

sorride un poco imbarazzata)

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Il controllo semiotico

Schoenfeld (1985): **control** in problem solving activities deals with "global decisions regarding the selection and implementation of resources and strategies" (p. 15).
It entails actions such as: planning, monitoring, assessment, decision-making, and conscious meta-cognitive acts.

Semiotic control: the decisions concern mainly the selection and implementation of semiotic resources (Arzarello & Sabena, 2011)

Semiotic resources: according to a multimodal perspective (language, written signs, gestures, …)

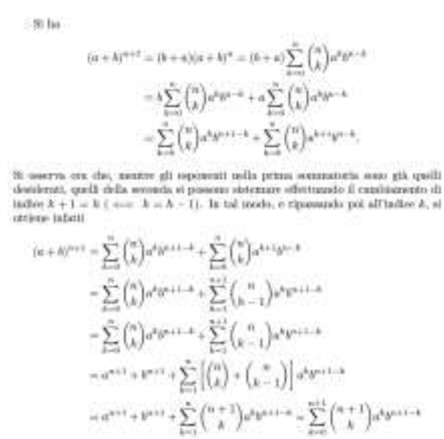
How this control allows to describe the processes of the students and to point out differences between the two excerpts.

Compito 4

Guarda la seguente dimostrazione, usuale nei corsi di Matematica:

- Quali difficoltà presenta?
- Quali tecniche pensi si possano usare con gli allievi per superarle?

Scrivi al massimo in una pagina le tue osservazioni.



Si ha

$$\begin{aligned}(a+b)^{n+1} &= (b+a)(a+b)^n = (b+a)\sum_{k=0}^{n}\binom{n}{k}a^k b^{n-k}\\ &= b\sum_{k=0}^{n}\binom{n}{k}a^k b^{n-k} + a\sum_{k=0}^{n}\binom{n}{k}a^k b^{n-k}\\ &= \sum_{k=0}^{n}\binom{n}{k}a^k b^{n+1-k} + \sum_{k=0}^{n}\binom{n}{k}a^{k+1} b^{n-k}.\end{aligned}$$

Si osserva ora che, mentre gli esponenti nella prima sommatoria sono già quelli desiderati, quelli della seconda si possono sistemare effettuando il cambiamento di indice $k+1=h$ ($\iff k=h-1$). In tal modo, e ripassando poi all'indice k, si ottiene infatti

$$\begin{aligned}(a+b)^{n+1} &= \sum_{k=0}^{n}\binom{n}{k}a^k b^{n+1-k} + \sum_{k=0}^{n}\binom{n}{k}a^{k+1} b^{n-k}\\ &= \sum_{k=0}^{n}\binom{n}{k}a^k b^{n+1-k} + \sum_{h=1}^{n+1}\binom{n}{h-1}a^h b^{n+1-h}\\ &= \sum_{k=0}^{n}\binom{n}{k}a^k b^{n+1-k} + \sum_{k=1}^{n+1}\binom{n}{k-1}a^k b^{n+1-k}\\ &= a^{n+1}+b^{n+1}+\sum_{k=1}^{n}\left[\binom{n}{k}+\binom{n}{k-1}\right]a^k b^{n+1-k}\\ &= a^{n+1}+b^{n+1}+\sum_{k=1}^{n}\binom{n+1}{k}a^k b^{n+1-k} = \sum_{k=0}^{n+1}\binom{n+1}{k}a^k b^{n+1-k}\end{aligned}$$

Approfondimento

Leggi i lavori di
- J. Mason: *Beeing mathematical with and in front of learners; A phenomenal approach to mathematics.*
- L. Radford: *The eye as a theoretician*

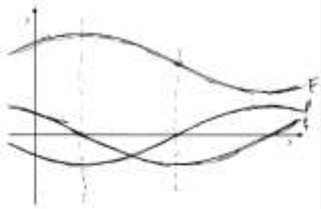
Commenta ora i tuoi processi di pensiero nella soluzione dei due esercizi (Formula e Induzione) usando le due lenti fenomenologiche (quando e se del caso) proposte da Mason e Radford e commentando se, quando, e come avviene una qualche forma di disclosure (chi ha questo tipo di interesse può approfondire studiando il volume di G.C. Rota, *The end of Objectivity*).

Compito 5

Mentre studiavano per l'esame di matematica, Lorenzo e Francesca si sono imbattuti nel seguente problema, trovato in un libro di testo:

Problema:
Il disegno qui sotto mostra un grafico con tre funzioni:
- Una funzione *f*
- La derivata di *f*
- Una primitiva di *f*

Identifica il grafico di ogni funzione.



Problema 0

Problema 1

Lorenzo e Francesca hanno bisogno del tuo aiuto! Vogliono sapere come risolvere il problema che hanno trovato nel libro di testo. Più che accontentarsi soltanto della risposta però, vogliono un *metodo* da usare per risolvere altri problemi come questo, in modo da essere preparati per domande simili all'esame.

Il tuo compito:
Pensa ad un metodo che Lorenzo e Francesca possono usare per risolvere problemi come quello trovato nel libro. Il tuo metodo deve funzionare non solo per tale problema, ma anche per problemi simili a questo, come quelli delle schede successiva. Scrivi una lettera a Lorenzo e Francesca in cui
(1) descrivi il tuo metodo,
(2) spieghi perché funziona (eventualmente sotto quali ipotesi),
(3) mostri come usarlo per risolvere il problema 0 e i successivi problemi 2, 3, 4.

Usate gli strumenti semiotici introdotti nel corso per interpretare/commentare:
- il fimato (minuti 8 – 10)
- il testo prodotto (confrontate con 40:15-42:15 oppure 1:13-1:15)

Compito 6

Sum of the first n numbers - 1 = 1/2n(n+1) -1

Sum of the first ½ n(n+1) odd numbers = ¼ n^2 $(n+1)^2$

$\boxed{\sum_{1}^{n-1}(j+1)}$

(k+1)-th odd number

$$\sum_{k=\sum_{1}^{n-1} j}^{\sum_{1}^{n-1}(j+1)} \boxed{(2k+1)} = n^3$$

$\boxed{k=\sum_{1}^{n-1} j}$

Sum of the first ½ n(n-1) odd numbers = ¼ n^2 $(n-1)^2$

Sum of the first (n-1) numbers = 1/2n(n-1)

$\frac{1}{4} n^2 (n+1)^2 - \frac{1}{4} n^2 (n-1)^2 = \frac{1}{4} n^2 (n^2 + 1 + 2n - n^2 - 1 + 2n) = \frac{1}{4} n^2 \, 4n = n^3$

multimodality

QUESTION 2

Solve the Problem 2 in a small group: one participant observes what happens (15 m)

Discuss in the group the following items (25 m):

What resources are used by the solvers?

Use the Semiotic Bundle to interpret what happens in the episode of the video.