Summary

Since longtime mathematics educators have shown interest in the use of history of mathematics in mathematics teaching. In many countries curricula mention the need of introducing a historical dimension in mathematics teaching. This chapter discusses some interesting reasons put forwards by the supporters of this use and their epistemological assumptions.
The initial part of the chapter provides a short account of the setting in which the first discussions and the first experiments concerning the use of history in mathematics teaching took place. Afterwards the chapter outlines the development of the community of scholars interested in the relationship between history and pedagogy of mathematics, which in 1976 was officially established as the group HPM (History and Pedagogy of Mathematics) affiliated to ICMI (International Commission on Mathematical Instruction). Some materials produced in this context are reported in the Appendix. They constitute a background and a source for researchers and for mathematics teachers wishing to explore the opportunity offered by history in their teaching.

About the introduction of history of mathematics in the classroom the chapter focuses on two main streams of action: - history for promoting the image of mathematics as a vivid discipline with links with reality, - history for dealing with mathematical concepts.

An efficient introduction of history in teaching entails adequate teachers’ historical knowledge. Then a part of this chapter is dedicated to discuss the role of this knowledge and to present how history may be used in teacher training programs.

At the end of the chapter some frequent objections put forwards by teachers about the possibility of introducing history in their teaching are presented. The conclusion is that, tough there are difficulties and some contexts are not favorable to this introduction, in suitable contexts the effort required for facing this endeavor will be rewarded by significant improvements in the classroom life.

1. Introduction

Many educators recognize that history of mathematics may have a role in mathematics education. The arguments that support this opinion are various. It is claimed that using history makes mathematics be perceived as a human endeavor, that it allows seeing the multiple facets of concepts and theories, and highlighting obstacles met in mathematical understanding. Moreover, together with epistemology, history of mathematics is considered suitable for setting mathematical objects in specific problematic contexts: evolution of rigor, ideologies, methods, forms of discourse, and links with other disciplines.

In this chapter the role of history of mathematics in mathematics education is discussed through theoretical considerations and a few examples of practice in the classroom and in teacher training. A brief historical survey introduces to the theme by showing that since longtime history of mathematics in mathematics education is a theme that has interested mathematicians and educators.

2. The Pioneer Period in the Introduction of History in Mathematics Education
2.1. The Scenario

In the second half of nineteenth century, when old states were modernized and new ones were established, one of the main concerns of the governments was to update or create systems of education in their countries. To this aim all school levels were considered with different objectives and approaches. For the primary level the main problem was the literacy of the population, for more advanced grades it was that of deepening the students’ background and to prepare for professions. In the meanwhile mathematical research was developing in many domains.

As always, mathematics was a main topic in curricula and very soon the problem of improving mathematics education became a theme of discussion. At the beginning this discussion was mainly carried out by mathematicians. Slowly mathematics teachers, who were acquiring a defined professionalization, entered the discussion and, in some cases, experienced some innovations in their classrooms. This mainly happened at the end of nineteenth century, when associations of mathematics teachers, journals addressed to mathematics teaching, new mathematics textbooks appeared in various countries, see (Furinghetti, to appear). Later on cooperation in mathematics education developed at an international level thanks to the creation in 1908 of the “International Commission on the Teaching of Mathematics” which may be considered the parent of the present ICMI (International Commission on Mathematical Instruction), see (Furinghetti & Giacardi, 2008).

Among the means considered for enhancing mathematics education there was history of mathematics. Since the eighteenth century important treatises on history of mathematics had been published and later some treatises, such as *A Short account of the history of mathematics* by Walter William Rouse Ball published in London (1888), put at disposal of a large audience historical knowledge. Journals devoted to history of mathematics were founded in the second half of nineteenth century: the Italian *Bulettnino di Bibliografia e di Storia delle Scienze Matematiche e Fisiche* (founded in 1868 by Baldassarre Boncompagni), the German *Abhandlungen zur Geschichte der Mathematik* (published in 1877 by Moritz Cantor), and the Swedish *Bibliotheca Mathematica* (founded in 1884 by Gustaf Hjalmar Eneström, see Lorey, 1926). Chapter 1 of the ICMI Study volume on “History in mathematics education” reports passages, taken from different epochs and countries, that show the widespread concern about the value of history of mathematics in the mathematical culture, see (Fasanelli et al. 2000). The oldest quotation, dating back to the 1790s, is authored by the outstanding mathematician Joseph Louis Lagrange, who stresses the importance of history for mathematical researchers.

2.2. Pioneer Reflections on the Use of History in Mathematics Education

The development of interest in history of mathematics was soon paralleled by the feeling that knowledge on history of mathematics may have a role in the teaching and learning of mathematics. A document that may be considered an archetype of the discussion about the use of history in mathematics education is the text of the talk
delivered by G. Heppel (1893) at the Association for the Improvement of Geometrical Teaching, the association parent of Mathematical Association, the British association of mathematics teachers. In his article Heppel recalls that historical information appears in treatises of mathematics recently appeared, that some teachers use historical illustrations and that he is using history with private pupils. Before explaining the benefits of the use of history in mathematics teaching, he lists the most important restrictions in this use:

I. The History of Mathematics should not form a separate subject of education, but be strictly auxiliary and subordinate to Mathematical teaching.

II. Only those portions should be dealt with which are of real assistance to the learner.

III. It is not to be made a subject of examination. (Heppel, 1893, pp. 19-20)

To answer the main question “in what ways History makes mathematical study easier, clearer, or more interesting” (p. 22) first of all Heppel remarks that history “gives us stereoscopic views instead of pictures and diagrams. A particular subject may be looked at from many sides, each aspect suggesting a different mode of treatment.” (p. 22) Afterwards he considers that through history some row ideas of a concept that has been covered by the successive developments may be highlighted to help the full understanding of this concept. Another benefit is that history contrasts the common feeling that mathematics is a dry subject, by recovering the cultural value of mathematics: “Mathematics is full of life and interest, that it appeals to the imagination as well as to the intellect, that it has a poetry peculiarly its own.” (p. 24) Moreover history of mathematics shows how progress in mathematics “has gone on in answer to the needs that men have felt” (p. 24). As shown in the following, the arguments mentioned by Heppel (1893) are present in the discussion on history in mathematics education until our days.

Heppel’s address mainly refers to pupils of primary and secondary levels. Other authors of that period consider also advanced levels. In the title of a paper published in 1899 by the Italian historian of mathematics Gino Loria, history of mathematics is seen as a “coupling link” between secondary and university teaching because it may help to revisit mathematics from an advanced standpoint, see (Furinghetti, 2000). Considering tertiary level involves, in particular, considering mathematics teacher education. In this concern Florian Cajori wrote in the introduction of his seminal book A history of mathematics: “Another reason for the desirability of historical study is the value of historical knowledge to the teacher of mathematics.” (p. 3). Another famous book on history of mathematics, written by Hieronymus Georg Zeuthen, was intended for teachers and proposed that the history of mathematics should be part of teachers’ general education, see (Zeuthen, 1902). This opinion was shared by mathematicians and educators. In 1904 the third International Congress of Mathematicians, held in Heidelberg, adopted a motion in which the introduction of a historical component in public education as well as the teaching of history of exact sciences in university courses were advocated, see (Krazer, 1905). The theme of history in teacher education permeates the work of the educator David Eugene Smith: the course for mathematics
teachers he held at the Michigan State Normal School in Ypsilanti was strongly based on the historical perspective (Donoghue, 2006) and his famous book *The teaching of elementary mathematics* is imbued with history of mathematics, see (Smith, 1904). Historical sections appear in *Elementary mathematics from an advanced standpoint*, the work where the mathematician Felix Klein presents the mathematical content he considered necessary for mathematics teachers, see (Klein, 1939). In part II (*Geometry*), he writes:

*I shall draw attention, more than is usually done ... to the historical development of the science .... I hope, by discussions of this sort, to further, as I like to say, your general mathematical culture: alongside of knowledge of details, as these are supplied by the special lectures, there should be a grasp of subject-matter and of historical relationship [emphases in the original] (II, p. 2).

The previous short outline of the pioneer discussion on the use of history in mathematics education stresses the existence of two domains of action: student education and teacher education. Though there are obvious links and some theoretical frames are common, I’ll treat separately the two domains and I’ll provide some specific examples for both.

![Branford’s schema about the mathematical stages of civilizations and of students’ education](image)

Figure 1. Branford’s schema about the mathematical stages of civilizations and of students’ education

### 2.3. A Pioneer Experiment of Introducing History in the Mathematics Classroom

A telling evidence of an early attempt in the use of history is provided by the text of the talk presented at the *Mathematical Association* in 1913 by Miss Barwell, who accounts her introduction of history of mathematics to students of the Training Department of Alexandra College (Dublin) and to girls aged sixteen and seventeen of other classes. The author’s very words illustrate the cultural climate and aims of this experiment (Barwell, 1913, p. 72):

*While reading for these lectures, I was greatly struck by the stress laid by Benchara Branford in his Study of Mathematical Education on the importance of what one might call the historical method. He emphasises the fact that the history of each individual development is a brief compendium of the history of the race, and that the sound method of instruction is to let the student travel, in his quest for knowledge, roughly*
over the same path by which his fathers arrived, - roughly, only, because life is short, and there were quagmires in which our fathers floundered for many centuries.

I thought it would be very good for the training-students to learn a little of how “Mathematics” grew, before they studied how to teach them, and so I sacrificed a certain amount of their very limited time to this object. And I was glad to find how much their interest was stimulated - especially among those who knew a little mathematics, - and though it was barely possible to do more than stimulate interest, one hopes that some of them will care enough to read more of the subject for themselves when the brief fever of training is at an end.

![Figure 2. Miss Barwell’s adaptation of Branford’s schema](image)

In the classroom Miss Barwell treated notations, Egyptian mathematics, systems of numeration, positional notations, the rise of algebra and of geometry. The works she cites in her talk suggest that she was a very special teacher, well acquainted about literature and academic events. The book she mentions in the above quotation is (Branford, 1908), a famous treatise on mathematics teaching that presented many innovative methods of teaching, such as the use of manipulatives and mathematical laboratories, as well as issues outside mathematics such as psycho-analytical theories, in particular the function of sub-consciousness. With this book Branford launched a program of empirical research in mathematics education in years in which there was almost no empirical research. Branford makes often reference to history: his book, in particular, contains a figure outlining the parallelism between mathematical development in the civilizations and individuals’ stages of learning, see Figure 1. Branford’s use of history has been considered as a form of application of recapitulation (ontogeny recapitulates the phylogeny), see (Fauvel, 1991). Schubring (2006) argues this position and claims “In view of the absence of empirically confirmed propositions concerning the process of learning in mathematics, Branford’s approach may be understood as using history of mathematics as a guideline for formulating research questions which then have to be investigated empirically.” In (Barwell, 1913) (p. 332) there is an adaptation of the figure conceived by Branford, see Figure 2.

3. Convergences and Divergences between Historical Conceptual Developments and Classroom Learning in Mathematics. Epistemological Assumptions about the Relation between Students’ Understanding and History of Mathematics
3. Convergences and Divergences between Historical Conceptual Developments and Classroom Learning in Mathematics. Epistemological Assumptions about the Relation between Students’ Understanding and History of Mathematics

In 1874 the German scientist Ernst Heinrich Haeckel formulated his law on the recapitulation. This is his famous statement (Haeckel, 1912):
The series of forms through which the individual organism passes during its development from the ovum to the complete bodily structure is a brief, condensed repetition of the long series of forms which the animal ancestors of the said organism, or the ancestral forms of the species, have passed through from the earliest period of organic life down to the present day. (pp. 2–3)

The law of recapitulation was transposed from biology to psychology (by Haeckel himself) through the claim that in their intellectual development, students naturally traverse more or less the same stages as mankind once did. As a consequence the functioning of the children psyche was considered as something traveling along the same path as their ancestors. This was taken as an argument for introducing a historical perspective in mathematics education at the start of the twentieth century. The views on education by Herbert Spencer were influencing the acceptance of this argument. Branford himself quotes Spencer’s claim that “the education of the child must accord, both in mode and arrangement, with the education of mankind, considered historically” (Branford, 1908, p. 326). Later on Haeckel’s theory revealed some flaws both from the biological and the psychological points of view. Nevertheless, recapitulation with its different variants entered the discourse in mathematics education, as shown by the work of the psychologists Jean Piaget and Lev Semënovič Vygotskij, see (Furinghetti & Radford, 2002; 2008). The relation between phylogeny and ontogeny has recently been subject to a deep revision and new reflections were carried out with the goal of finding more suitable explanations concerning it. This was fostered by the emergence of new conceptions about the role of culture in the way we come to know and think. Radford (1997) and Schubring (2006) have drawn attention to the role played by conceptions about history in shaping the view of the relation between biological and psychological recapitulation. Historians of mathematics and epistemologists have shifted the attention to the discussion on the very nature of the history of mathematics, see (Fried, 2001; Grattan-Guinness, 1973; 2004).

The relation between history of mathematics and mathematical thinking has interested also outstanding mathematicians. The following sentence by Poincaré (1899) “Zoologists claim that the embryonic development of animals summarizes in a very short time all the history of its ancestors of geologic epochs.” [Les zoologistes prétendent que le développement embryonnaire d’un animal résume en un temps très court toute l’histoire de ses ancêtres des époques géologiques.] (p. 159; my translation) evidences that the mathematician Henri Poincaré was aware of the problem of recapitulation. His position is close to Klein’s view on the effectiveness of history in mathematics teaching. Klein used history in his courses for prospective teachers, but he was not applying the recapitulation law in a literal sense. For him, as for Poincaré, history was a way to abolish the use of mathematical logic and the excesses of rigor, as advocated by some of their colleagues in those years. Consequently he supported the genetic ordering of the teaching subjects as opposed to the traditional “systematic” teaching. Many decades later Thom (1973) expressed a similar feeling in his criticism to the movement of New/Modern Mathematics.

Another approach to teaching linked to history of mathematics is the so-called genetic approach. The term “genetic” is an ambiguous one, because it is used with different meanings. The meaning here is the one given by Otto Toeplitz in the description of his
method used for teaching analysis to university students. In the introduction of his manual on calculus Toeplitz (1963) writes: “If we were to go back to the origins of these ideas [basic topics in infinitesimal calculus which we teach today as canonical requisites], they would lose that dead appearance of cut and dried facts and instead take on fresh and vibrant life again.” A complete study of the genetic method as envisioned by Toeplitz can be found in Schubring (1978).

Burn (1999) explains Toeplitz’s ideas in the following way:

_The question which Toeplitz was addressing was the question of how to remain rigorous in one’s mathematical exposition and the teaching structure while at the same time unpacking a deductive presentation far enough to let a learner meet the ideas in a developmental sequence and not just in a logical sequence. While the genetic method depends on careful historical scholarship it is not itself the study of history. For it is selective in its choice of history, and it uses modern symbolism and terminology (which of course have their own genesis) without restraint._ (p. 8)

In looking for an approach to calculus at the students’ grasp Burn (1993; 1999; 2005) resorts to the genetic method. Also Farmaki and Paschos (2007) use a genetic approach for introducing concepts of calculus. All these approaches reflect the authors’ belief that the students’ appropriation of mathematical knowledge is a process that requires their active engagement with the objects of knowledge.

Freudenthal (1973) provides an interpretation of the genetic method through his method of “guided reinvention:”

_Urging that ideas are taught genetically does not mean that they should be presented in the order in which they arose, not even with all the deadlocks closed and all the detours cut out. What the blind invented and discovered, the sighted afterwards can tell how it should have been discovered if there had been teachers who had known what we know now.... It is not the historical footprints of the inventor we should follow but an improved and better guided course of history._ (pp. 101, 103; my italic)

Van Amerom (2003) has applied Freudenthal’s interpretation of the genetic method in her project “Reinvention of algebra,” which uses “informal, pre-algebraic methods of reasoning and symbolizing as a way to facilitate the transition from an arithmetical to an algebraic mode of problem solving” (p. 73).

Also Radford and Guérette (2000) claim that their experiment leads students to reinvent the formula that solves the general second-degree equations. They use old Babylonian texts studied by the historian Jens Høyrup and some manipulatives.

The scholars’ struggle about the relationship between contemporary students’ construction of mathematical knowledge and the historical construction of mathematical knowledge evidences the need of clarifying this relationship when designing classroom activities (for students or for teacher training). As noted by Radford, Boero and Vasco (2000), this clarification concerns on the one hand the psychological domain (as for learning processes of contemporary students) and on the other hand the historico-epistemological domain (together with other domains).
According to the different set of epistemological assumptions, there are different perspectives. The most widely known are the “epistemological obstacles” and the socio-cultural perspectives. The idea of epistemological obstacles was developed by Gaston Bachelard in the 1930s and later on imported into didactics of mathematics by Guy Brousseau in the 1970s, see (Brousseau, 1983). In Brousseau’s view there are some basic assumptions. The first is that epistemological obstacles pertain to the sphere of knowledge, which for Brousseau is separated from other spheres. The second assumption is that epistemological obstacles are characterized by their reappearance in history of mathematics and in the present learning of mathematics. The third epistemological assumption relies in the articulation “student/milieu.” According to Brousseau the development of knowledge is a sequence of conceptions and obstacles to be overcome, with obvious consequences on the organization of teaching situations. The socio-cultural perspective moves from the epistemological assumptions of Vygotskij’s approach in mathematics education. Radford has elaborated this perspective in numerous works, see (Radford, 1997) and successive papers by this author. He assumes that knowledge is socially constructed. Following Vygotskij he refers to interiorization when considering the way in which individuals appropriate the cultural knowledge of their culture. The interiorization is not seen as a passive process, but as an active one, in which individuals create concepts and meanings through the use of signs and discourse.

4. International Cooperation in the Studies on the Use of History in Mathematics Education

The history of international cooperation in mathematics education has an important landmark in the foundation of the commission which is a direct parent of the present ICMI. This happened in Rome during the fourth International Congress of Mathematicians in 1908. In spite of the activity of this Commission at international level, no specific regular international conferences on mathematics education were held until 1969. In this year ICMI launched the tradition of having a periodical International Congress on Mathematical Education (ICME). These congresses, which after the second in 1972 became quadrennial, are an international occasion for presenting experiments, innovations, and themes of interest in mathematics education. Already at ICME-2 (1972, Exeter, UK) a Working Group on “History and pedagogy of mathematics” was organized. The work of this group was continued in the following ICME-3 (1976, Karlsruhe, Germany), and in the same year the Executive Committee of ICMI approved the affiliation of the new Study Group under the title “International Study Group on Relations between History and Pedagogy of Mathematics, cooperating with the International Commission on Mathematical Instruction”, now generally shortened to “HPM”. The history of the first 25 years of the group HPM is outlined in the section "The Affiliated Study Groups" in (Furinghetti & Giacardi, 2008). General information about the activities of HPM and the Newsletter are in HPM website http://www.clab.edc.uoc.gr/HPM/).

The “principal aims” of the Study Group were given in these words (Historia Mathematica, 1978, 5, p. 76):

http://greenplanet.eolss.net/EolssLogn/mss/C02/E6-132/E6-132-65/E6-1...Relation_between_Students'_Understanding_and_History_of_Mathematics
1. **To promote international contacts and exchange information concerning:**
   
   a. Courses in History of Mathematics in Universities, Colleges and Schools.
   b. The use and relevance of History of Mathematics in mathematics teaching.
   c. Views on the relation between History of Mathematics and Mathematical Education at all levels.

2. **To promote and stimulate interdisciplinary investigation by bringing together all those interested, particularly mathematicians, historians of mathematics, teachers, social scientists and other users of mathematics.**

3. **To further a deeper understanding of the way mathematics evolves, and the forces which contribute to this evolution.**

4. **To relate the teaching of mathematics and the history of mathematics teaching to the development of mathematics in ways which assist the improvement of instruction and the development of curricula.**

5. **To produce materials which can be used by teachers of mathematics to provide perspectives and to further the critical discussion of the teaching of mathematics.**

6. **To facilitate access to materials in the history of mathematics and related areas.**

7. **To promote awareness of the relevance of the history of mathematics for mathematics teaching in mathematicians and teachers.**

8. **To promote awareness of the history of mathematics as a significant part of the development of cultures.**

These aims describe the various activities linked to the theme of history in mathematics education. Since its foundation HPM has been a direct or indirect catalyst and instigator of initiatives such as regional and national conferences, publications of books and special issues of journals, creation and maintenance of websites. In the Appendix an abridged list of the most known initiatives and publications is given.

5. **History of Mathematics in Mathematics Education: Why, How, for Whom, When?**

There have been many attempts to answer the questions put forwards in the title of this section. The mother question is “Why to use history in mathematics teaching”. A first obvious answer is “to improve teaching/learning of mathematics.” Then the successive question should be “In which way the use of history contributes to the improvement?” or, equivalently, “Which are the functions of history in the teaching/learning process?” Discussions on these points and attempts of classification are present in literature, e.g. see (Jankvist, 2009; Tzanakis & Thomaidis, 2011). Here I take as a starting point a rough distinction of the goals in using history of mathematics in mathematics education according to two main streams (which, of course, are fuzzy at the edges and overlap in
many situations), see also (Furinghetti, 1997):

A. History for acting on the image of mathematics against some common myths about it and the negative attitude of some people towards mathematics. An important expected outcome is to recover the value of mathematics seen as a part of cultures through the reflection on the nature of mathematics as a socio-cultural process and as a human endeavor. The historical perspective is a way of living the mathematical experience.

B. History for treating mathematical concepts and processes. The expected outcome is fostering the appropriation of meaning of mathematical concepts.

The final goal of both streams is to make efficient and less arduous teaching and learning mathematics, to enrich the mathematical culture, and to give a wider meaning to the concept of culture itself. The two streams are not competing; in some cases they are complementary. It may happen that to have a rich image of mathematics stimulates the wish of understanding concepts and, vice versa, to understand concepts may influence the image of mathematics.

The aims and effects which might be pursued by way of historical activities may be described by three general ideas expressed in the chapter 9 of the ICMI Study volume on History in mathematics education. In this chapter Jahnke et al. (2000) discuss the potentialities of the use of original sources, but their arguments may be extended to the general situation of introducing a historical perspective in mathematics teaching. The notions of replacement, reorientation and cultural understanding are introduced as follows (pp. 291-292):

(i) Replacement

Integrating history in mathematics replaces the usual with something different: it allows mathematics to be seen as an intellectual activity, rather than as just a corpus of knowledge or a set of techniques.

(ii) Reorientation

Integrating history in mathematics challenges one’s perceptions through making the familiar unfamiliar. Getting to grips with a historical text can cause a reorientation of our views. History of mathematics has the virtue of ‘astonishing with what comes of itself’ (Veyne 1971). All too often in teaching, what happens is that concepts appear as if already existing. This is true for the concept of a set, for example, but just as true for the concept of a triangle or a function. And concepts are manipulated with no thought for their construction. History reminds us that these concepts were invented and that this did not happen all by itself.

(iii) Cultural understanding

Integrating history of mathematics invites us to place the development of mathematics in the scientific and technological context of a particular time and in the history of ideas and societies, and also to consider the history of teaching mathematics from
perspectives that lie outside the established disciplinary subject boundaries.

The first two notions are related to the teaching and learning of mathematical concepts and processes (Stream B). As a matter of fact to walk in the foreign and unknown landscape provided by history forces us to look around in a different manner and brings to light elements which otherwise would escape. In such a context one better grasps the roots around which the mathematical concepts were built over the centuries. Through reorientation the learners involved in the process are forced to find their own path towards the appropriation of meaning of mathematical concepts. The third notion emphasizes the role of history in shaping the cultural view of mathematics and the links with other disciplines (Stream A).

In the case where original sources are used in the classroom Jahnke et al. (2000) single out specific benefits. The first obvious benefit is the historical fidelity to the authors’ thinking. As discussed in Section 5.2, the historical fidelity allows to approach in a deep way the roots around which the mathematical concepts develop. Jahnke (1994) and Demattè and Furinghetti (2011) argue that the use of original sources may be a means for introducing students to hermeneutics. The original sources offer an additional value in the way they allow to promote contacts with languages, figures, forms of communication, and types of situations. From these elements the context assumes vivid connotations and stimulates socio-cultural reflection on present and past civilizations.

The original sources are a mine of insightful problems suitable for any school levels and for teacher training. Swetz (1995) considers several particular pedagogical techniques contained in ancient texts from Babylonian and Chinese civilizations: the use of an instructional discourse; a logical sequencing of mathematical problems and exercises and employment of visual aids. This author claims that the historical texts tell us much about how mathematical concepts and techniques were conceived and evolved. But they can also tell us even more: usually, their contents embody a pedagogy, specific organizational forms and methods used for teaching.

Unfortunately arranging a course around original sources presents some difficulties that are often mentioned by teachers who consider the possibility of using history. Siu (1995) explains how he resolved these difficulties by using excerpts available in the various source books written in the English language.

Sections 5.1 and 5.2 discuss separately the streams A and B and provide some examples and references. Further information about the introduction of history at different school levels is supplied by the bibliography and the appendix. An overall idea on the existing experiments in the case of geometry is provided by the survey presented in (Gulikers & Blom, 2001).

The following list of the ways of using history in (Fauvel, 1991, p. 4) mirrors rather exhaustively the activities emerged in the various experiments carried out in different contexts:

- Mention past mathematicians anecdotally
- Provide historical introductions to concepts which are new to pupils
- Encourage pupils to understand the historical problems to which the concepts
they are learning are answers

- Give "history of mathematics" lessons
- Devise classroom or homework exercises using mathematical texts from the past
- Direct dramatic activity which reflects mathematics interaction
- Encourage the creation of posters displays or other projects with a historical theme
- Setting projects about local mathematical activity in the past
- Using critical examples from the past to illustrate techniques or methods
- Explore past misconceptions/errors/alternative views to help in understanding and resolving difficulties for today’s learners
- Devise the pedagogical approach to a topic in sympathy with its historical development
- Devise the ordering and structuring of topics within the syllabus on historically-informed ground.

1. Introduction

5.1. Stream A: History for Promoting Mathematics
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One of the problems in mathematics education is to motivate students. Often it is claimed that to motivate students it is needed to “humanize mathematics”, see (Brown, 1996). The expression “humanize mathematics” has not a clear meaning and its explanation involves not only mathematicians, but also philosophers and epistemologists. At the International Mathematical Congress of Kyoto, Manin (1991)
stated that he wanted “to restore a certain balance between the technological and the humanitarian sides of mathematics.” (p. 1666). In the plenary talk delivered at ICME-7 (Québec) Tymoczko (1994) pointed out that philosophers were wrong in neglecting the real world as a basis of mathematics, because this discipline is an integral part of the “common sense”. For him mathematics has meaning as an activity of a community and seeing pure mathematics as the essence of mathematics has distorted the philosophical perspective on it. According to Tymoczko (1994), one may think of a civilization without pure mathematics: in this concern he mentions Egyptians and Babylonians.

Referring to the project launched by Alvin White (Harvey Mudd College, USA) who wanted mathematics taught in a human way, Tymoczko (1994) answers to the question “What made mathematics one of the humanities?” in this way:

Certainly not the mere fact that humans did it? Human do science too. [...] Pure mathematics is ultimately humanistic mathematics, one of the humanities, because it is an intellectual discipline with a human perspective and a history that matters. There is no answer to the question: What is important in mathematics, once for all? We can only ask what is important in mathematics to human beings, with given abilities and limitations at a given point in their mathematical development.” (Tymoczko, 1994, p. 334)

Tymoczko acknowledges that philosophers were wrong in considering only pure mathematics, but at the same time he ascribes to educators the opposite mistake of concentrating too much on useful mathematics. For Tymoczko this approach does not introduce students to the mathematical discipline or, better, to humanistic mathematics; his idea is that “To introduce students to humanistic mathematics is to introduce them to a human adventure, an adventure that humans have actually partaken of in history.” (ibidem, p. 335).

In mathematics teaching the use of history to humanize mathematics (in the different meanings the word humanize may have for different persons) is realized in various ways. Often anecdotes, stories, illustrations, form of dramatization, and vignettes are considered good devices for reaching the goal of humanizing mathematics. Not only schoolteachers, but also lecturers in graduate schools and universities use these devices. The justification for this use resides in the affective factors, which intervene in teaching and learning process. Firstly, these activities present mathematicians as persons: this fact makes closer mathematics world to students’ experience. Secondly, this use may foster communication and interaction in the classroom; it gives the chance to participate in the classroom life to students with different attitudes and abilities. One of the affective factors behind these activities is the personal pleasure in dealing with history felt by some teachers. In carrying out these activities teachers are satisfied and feel they have reached the aim of humanizing mathematics, see (Bidwell, 1993).

There are many interesting examples that show how dramatization may be an efficient tool in the repertoire of teachers wishing to deal with different aspects of mathematics. Gavin Hitchcock has produced dramatic pieces for humanizing and contextualizing the development of mathematical concepts, see, for example (Hitchcock, 1992). Other dramatic pieces of mathematics concern biographies of mathematicians, see (Ponza,
1998). Often students are supposed to be the actors in these plays.

In the Second European Summer University held in Braga (24-30 July, 1996) it was presented an exhibition entitled “Images made by students about their image of mathematics”, see (Gargani, 1996), which is the result of a project developed with Italian students aged 18 of a Liceo Artistico, a type of school oriented to give an education in visual arts. In this kind of school mathematics is a compulsory subject, but it is quite neglected and disliked by students. The mathematics teacher was frustrated by this situation and felt the need to investigate on the relationship with mathematics his students had. It is not easy to do this, taking into account that they tend to hide their feelings to not be displeasing the teacher. To satisfy his need the teacher planned an activity lasting for two years. Initially students faced historical and epistemological problems of mathematics, took part in debates guided by the teachers, and carried out personal research on history of mathematics. The sources used were classical texts of history of mathematics and encyclopedias (at the time of the experiment web was not yet easily available in schools). After this introductory phase the students were asked to write their impressions and their ideas on the development of mathematics through the centuries. In addition the teacher decided to exploit the graphical and pictorial abilities acquired by the students thanks to the particular type of school they were attending. Then students were asked to prepare one of these objects by choice: - a poster for a hypothetical conference on mathematics, - a figure concerning history of mathematics, - the cover of a mathematical book. Each graphical product had to be accompanied by student’s notes explaining which ideas they wished to express with their figure, which artistic techniques they used and whether the style of particular artists had inspired them. The students’ graphical works have been photographed, digitalized and elaborated on the computer by means of specific graphic programs. The materials produced are very nice from the aesthetical point of view and are really effective for having a feeling on how students perceive mathematics and which issues of its history impressed them. In an indirect way the activity promoted historical knowledge and personal reflection on the nature of mathematics.

Percival (2001) has attempted to show the “humanist, ‘human-made’ side of arithmetic” (p. 21) to her students aged twelve and thirteen. This was realized in the classroom by linking history of mathematics and mathematics itself through a social approach and multicultural co-operation. The mathematical subject dealt with was elementary arithmetic. In other experiments this subject has been treated through ancient documents, but the artifactual approach used in this experiment appears to be extremely suitable to the author’s purpose: students make their own constructions of objects (such as Babylonian tablets) and documents imitating those presented to them, and use ancient calculating devices, albeit in modern reconstruction. Thus concrete activity parallels conceptual construction of the notion of numerical systems and their manipulation.

Another way of promoting/humanizing mathematics is to link it to the cultural context, in particular to the context where students live. Problems of navigation have been treated by Ransom (1995) and Dias (2008): both these authors teach in countries (UK and Portugal respectively) with a strong tradition of sailing. Kool (1992) worked with low-attaining students in The Netherlands using 16th century Dutch textbooks. These
teachers were successful in combining the cultural goal of embedding mathematics in students’ socio-historical context with the specific goal of introducing mathematical concepts and processes.

5. 2. **Stream B: History of Mathematics for Constructing Mathematical Knowledge**

It has been already pointed out that one important function of history in mathematics education is to foster understanding through an approach to the roots around which concepts developed. I see a strict relation between the roots of concepts identified through history and the cognitive roots of concepts, described by Tall (2003) as concepts which are (potentially) meaningful to the student at the time, yet contain the seeds of cognitive expansion to formal definitions and later theoretical development. Tall (2003) mentions three representational worlds: embodied, symbolic-proceptual, formal-axiomatic. These three worlds encompass different modes of operating, which Tall put in relation with Bruner’s modes of mental representation. The term “embodied” is used by Tall (2003, p. 4) “to refer to thought built fundamentally on sensory perception as opposed to symbolic operation and logical deduction. This gives to the term ‘embodied’ a more focused meaning in mathematical thinking”. The symbolic-proceptual world refers to the triad “concept, process acting on it, symbol as a pivot between the two”. In school the mathematical objects are often approached in the second, or, even worst, in the third world. About this fact Skemp (1969) claims that a purely logical approach provides only the final product of the mathematical discovery and does not generate in the learner the processes by which mathematical discoveries are made, so that such an approach teaches the mathematical thought, not the mathematical thinking. When mathematics is presented in a polished and finished way the learners (both in school and in teacher education courses) have difficulties in identifying the cognitive roots of concepts in the magma of processes, concepts, and rules they have at their disposal. The embodied world is a suitable place to start the appropriation of the meaning of mathematical objects; it is the world where the “somatic” understanding mentioned in Tang (2006) may happen.

An example of students’ bodily experience is the measurement of inaccessible heights starting from historical documents; see (Gulikers, 2002-03). This paper refers to a project aimed at teaching similarity to 12 years old students. To this purpose the author uses a Dutch book by Cardinael (1610) in which a method to measure the height of a tower with the aid of a mirror is described. Another method to measure inaccessible heights is based on the old instrument known as the cross staff described in the geometry book of Pierre de la Ramée translated into Dutch in 1622. In the paper the original texts of some problems are reported as well as the pupils’ assignments.

Gravemeijer and Doorman (1999) use history of mathematics to introduce the derivative in a calculus course. These authors try to avoid the “anti-didactical inversion”, a teaching phenomenon pointed out by Freudenthal (1973), in which the end results of the work of mathematicians are taken as the starting points for mathematics education. Then their introduction to derivative takes its point of departure primarily in mathematics as an activity, and not in mathematics as a ready-made-
system. To this aim Gravemeijer and Doorman (1999) use the studies on kinematics carried out in Oxford (Merton College) in the first half of the 14th century. These studies investigated velocity as a measure of motion and tried to find a description of the distance traversed by a body moving with a uniform accelerated motion. After the Oxford studies Gravemeijer and Doorman (1999) present the kinematics studies of Nicole Oresme and Galileo. In this way a bodily experience of movement is the first step in the introduction of calculus concepts.

Furinghetti and Somaglia (1997) report on an experiment carried out in high school aimed at introducing the difficult concept of the link between the derivative and the integral. To give students an intuitive approach based on geometry and visualization the teacher presents the re-elaboration reported in Figure 3 of the tenth lesson from Lectiones opticae et geometricae of Isaac Barrow (1674, London; second edition, t. I and II)

![Image](http://greenplanet.eolss.net/EolssLogn/mss/C02/E6-132/E6-132-65/E6-132-65-TXT-03.aspx#5.1_Stream_A_History_for_Promoting_Mathematics)

Figure 3. Worksheet on the relation between the derivative and the integral

According to the teacher’s opinion the geometrical discourse of Barrow explains the link between the derivative and the integral better than the theorems reported in students’ textbooks. This way of working mirrors the teacher’s confidence on sensory perception to construct meaning. The explanatory character of Barrow’s lesson is exploited also in (Flashman, 1996).

6. History of Mathematics and Teachers

Section 5 discusses how history may be used with students, in Section 6 the focus is on teachers and discuss how history may be used in mathematics teacher education.

6.1. The Role of History of Mathematics in Teacher Education

The Chapter 4 of the ICMI Study volume on History in mathematics education is dedicated to teacher training, see (Schubring et al., 2000). The authors note that, differently from what was claimed in 1904 at the Third International Congress of Mathematicians, in a more recent study Dijksterhuis (1962) considered history of mathematics not essential for the students of mathematics, though useful for teachers. He said nothing about the use of history as a means for developing teachers’ mathematical knowledge, he was rather thinking at history as a motivational factor in the classroom. Later on Freudendhal (1981) expressed the doubt whether teachers’ knowledge on the history of mathematics affects their understanding of mathematics. Arcavi, Bruckheimer, and Ben-Zvi (1982) argue that this doubt is unfounded, depending on the context where activities on history of mathematics take place. These
authors list the variables of the context (target audience, their mathematical background, the sources, the approach, the topics, etc.) that may influence the outputs of the introduction of history in teacher education.

As a matter of fact literature reports different appraisals of the role of history in mathematics teacher education. There is a general consensus on the fact that knowledge of history enriches the mathematical culture of teachers and endows teachers with a form of the “integrated knowledge” that Freudenthal (1981) considers convenient for a good teaching. Some studies pay particular attention to the link between history and mathematics teachers’ beliefs and attitudes about mathematics. Hsieh (2000) considers negative numbers, a topic problematic to be taught, and investigates the beliefs of in-service secondary mathematics teachers’ who attended a course on mathematics education. The author’ conclusion is that learning the history of negative numbers may change secondary mathematics teachers’ beliefs of the topic in question and its teaching. Moreover he points out that teachers may learn and retain the history of negative numbers when the learning environment allows group discussion. Philippou and Christou (1998) illustrate the results of a three-year program for prospective primary teacher training that included history of mathematics. The results indicated significant improvement of attitudes, particularly about the satisfaction from and the usefulness of mathematics. Charalambous, Panaoura and Philippou (2009) studied how prospective mathematics teachers’ attitudes and beliefs were influenced by a teacher training program grounded in the history of mathematics. These authors found mixed results, but recognized the potential for history of mathematics to foster the development of knowledge for teaching. In a course for prospective mathematics teachers Clark (2011) used historical material for discussing the teaching of algebra. According to this author, her students’ claims evidence the enrichment both in mathematical and pedagogical knowledge about quadratic equations.

Other studies draw different conclusions. Stander (1989) conducted an investigation on 31 countries and found considerable interest for history in Denmark, Norway and US. He has analyzed two experiments. In the first one 63 secondary students were divided in two groups: one group (the control group) was presented with a teaching package containing the mathematical material connected with Euler relation for convex polyhedra, the other group had the same mathematical material with additional material which was concerned with the discovery and proof of the relation. The second experiment concerned prospective primary schoolteachers. The entry of students to a university school of Education was divided in two groups. One group was given a weekly historical enrichment sheet about the mathematics they had been studying. The other group (the control group) was given no enrichment material. To measure changes in attitude when history was used in teaching mathematics an attitude questionnaire was used. In both cases no significant effect in changes of attitudes were observed. The author is aware that the short duration of the historical activity and the limitation of his experiment prevent to draw general conclusions. Fraser and Koop (1978) presented to 39 mathematics teachers a copy of a mathematical play on Thales and an article relating historical stories to be used in the classroom. Teachers’ opinions about the materials were generally favorable, but they were not ready to use them in their own teaching.
The previous studies stress the importance of investigating on teachers’ beliefs as a variable in the success of implementation of history in the classroom. Alpaslan, Isiksal, Haser (2011) have developed an instrument for determining pre-service mathematics teachers’ attitudes and beliefs towards using history of mathematics in school mathematics courses. Smestad (2011) describes an interview study on Norwegian teachers’ conceptions of the history of mathematics. From his findings the author concludes that one important reason for the poor results obtained when including the history of mathematics into the Norwegian curriculum is that teachers’ conceptions and knowledge of the history of mathematics were not taken into account. It seems that curriculum developers deemed that once history was included in the curriculum, teachers would know what to do, while Smestad’s findings show clearly that this idea is wrong.

6.2. An Example of Realization

The different conclusions drawn by the various authors on the influence of history in teacher education programs are linked to the methods and the goals of these programs. Once assumed that teachers must have some basic information about history, the design of courses must be strictly related to the professional needs of teachers. In the perspective of integrating historical and pedagogical goals in teacher training, successful courses, as those described in Arcavi, Bruckheimer and Ben-Zvi, (1982 and 1987), were based on historical materials aimed at enhancing teachers’ pedagogical knowledge on concepts and methods. The main goal of these courses was not just to provide teachers with historical knowledge, but rather to use history to promote the reflection on the ways of representing and formulating the concepts that make them comprehensible to their students.

To explain my ideas on this point I present an example of activity in my courses for prospective secondary teachers. My students are students in mathematics whose university curriculum is aimed at teaching. They hold beliefs that have been elaborated mainly on the ground of school experience. Thompson (1992) claim that these beliefs concern: the nature of mathematics and its concepts, self as a learner and as a teacher, the process of teaching and learning. Researchers in education ascribe importance to prospective teachers’ beliefs because they may influence the future teachers’ choices for the classroom. In my courses for prospective teachers I make my students challenge their existing beliefs for fostering flexibility and openness to different styles of teaching. As said before, history, if appropriately used, creates an environment suitable to reflection on mathematical concepts to be taught.

At secondary level algebra is one of the topics difficult to be taught. To challenge my students’ beliefs about it I use medieval arithmetic problems, see, for example, the following problem 47 taken from the medieval treatise Trattato d’Aritmetica by Paolo Dell’Abbaco (also known as Paolo Dagomari):

_A gentleman asked his servant to bring him seven apples from the garden. He said: “You will meet three doorkeepers and each of them will ask you for half of all apples plus two taken from the remaining apples.” How many apples must the servant pick if_
he wishes to have seven apples left?

The students are invited to solve this problem and to write carefully their solution process. Afterwards, they analyze the written process of a mate. At the end of the session, the students discuss the findings. In the following I present two examples of processes produced by my students that evidence the possibility of two ways of solving the problem:

Chiara’s solution. The student starts from the apples required before passing through the last door. Since the doorkeeper asks half plus 2 apples, the 7 apples are half of the amount less 2. Then, before the last door, the servant has 18 apples. The student observes that 18 is \( 7 \times 2 + 4 = 18 \), then, deduces that before the second door, the servant has \( 18 \times 2 + 4 = 40 \) apples and thus he must pick \( 40 \times 2 + 4 = 84 \) apples.

Grazia’s solution. The student names \( y \) the apples picked by the servant. Then apples left after meeting the first doorkeeper are \( (y - y/2) - 2 \). Repeating this reasoning after the meeting with the second and the first doorkeeper the student writes the algebraic model (a first degree equation in the unknown \( y \)) of the problem.

Two solving paths have been followed, which may be put in relation with the analytic (from the unknown to the known) and synthetic (from the known to the unknown) methods:

- Chiara’s arithmetic path: from the known (left apples) to the unknown (apples to be picked)
- Grazia’s algebraic path: from the unknown (apples to be picked) to the known (left apples).

Figure 4. Problem 47 in Paolo Dell’Abbaco’s *Trattato d’Aritmetica*

To illustrate the different nature of arithmetic and algebraic problems I propose further problems taken from the medieval manuscript *Propositiones ad acuendos juvenes* (*Problems to sharpen the young*) by Alcuin of York. Here is an example of these problems translated from Latin into English.

*Problem 4. A certain man saw some horses grazing in a field and said longingly: “O that you were mine, and that you were double in number, and then a half of half of this [were added]. Surely, I might boast about 100 horses.” Let him discern, he who wishes, how many horses did the man originally see grazing?*

With this activity the prospective teachers are led to reflect on the fact that algebra is not only generalization, not only abstraction, not only using symbols, not only an extension of arithmetic: algebra is a method and the analytic method is its core, see (Charbonneau, 1996). This way of looking at algebra was adopted by François Viète in
the sixteenth century and allowed him to clarify the different role of variables, parameters, and unknowns. His work is a landmark in the process of creating the new algebra, see (Hunger Parshall, 1988). The fact that the analytic method is the core of algebra was discussed by Furinghetti (2007) in another experiment based on the cut and paste method of Al-Khwarizmi for solving second degree equations.

The prospective teachers may use the medieval problems presented in the training course for their work in the classroom and exploit other potentialities, such as the reflection on language, the links with literature and with general history. In this perspective history generates also cultural understanding, since the development of mathematics is set in the context of a particular time and in the history of ideas and society.

3. Convergences and Divergences between Historical Conceptual Developments and Classroom Learning in Mathematics. Epistemological Assumptions about the Relation between Students’ Understanding and History of Mathematics

6.3. Resources, Prescriptions, Opportunities at Teachers’ Disposal for Introducing History of Mathematics in the Classroom
After having acknowledged the role of history of mathematics in teacher education, it is necessary to reflect on some key aspects of the setting in which teachers operate. In the national guidelines for mathematics teaching of many countries history of mathematic
is mentioned, but not always this suggestion is accompanied by structured plans and resources for preparing teachers to put it into practice. The national policies of teacher training differ from a country to another, as shown by the survey reported in the chapter 4 of the ICMI Study volume on History in mathematics education, see (Schubring et al., 2000), and, more recently, by a panel on this subject, see (Barbin, Furinghetti, Lawrence, & Smestad, 2011). In some countries there are national or local courses, compulsory or not, aimed at providing teachers with some basic knowledge on history of mathematics, but a lot of teachers still have not this basic knowledge.

Beside the programs of teacher training (when they exist), which tools are at teachers’ disposal for conceiving personal ideas and plans encompassing a historical dimension? Good treatises on history of mathematics help to elaborate general ideas on the historical development of mathematics. For those who have not regular access to historical libraries or are not fluent with ancient or foreign languages, the readers are a good source of original passages, translations and interpretation. At present web makes accessible a considerable amount of original sources. Moreover, there are special programs aimed at promoting the implementation of history of mathematics in mathematics teaching. English speaking teachers may plan teaching sequences thanks to the materials of the CD-Rom edited by Katz and Michalowicz (2004), which contains eleven historical modules to be used in the classroom. These modules cover different types of objectives and topics from pre-algebra to calculus. The Mathematical Association of America (MAA) maintains a section (called Convergence) of its website Loci (See http://mathdl.maa.org/mathDL/46/), which offers a wealth of resources to help teachers teach mathematics using its history. In France the net of the institutes of research in mathematics education IREM (Instituts de Recherche sur l’Enseignement des Mathématiques) produces historical materials that have been experimented in the classroom and that may be transferable to other situations. In the UK teachers have at disposal a good selection of articles, classroom ideas, and resources for the classroom on the history of mathematics provided by the two major mathematics education websites, the NRich (See http://nrich.maths.org/public/) and Plus Magazine (See http://plus.maths.org/content/).

The various experiments of introducing a historical dimension in mathematics teaching carried out by teachers show the importance of the production of materials that combine historical information with pedagogical aspects. In presenting her pioneer attempts of using history in teaching (see Section 2.3) Barwell (1913) explicitly mentions that the inspiration came to her from the books of mathematics education by Branford (1908) and by Smith (1904).

Considering that national guidelines in different countries advocate the introduction of a historical dimension in teaching, one would expect to find traces of these guidelines in the textbooks, namely to find historical materials and specific hints. The surveys presented in (El Idrissi, 2006) and in (Boyé, Demattiè, Lakoma, & Tzanakis, 2011) the ways history of mathematics appear in the textbooks may be roughly summarized according to the following patterns:

- Strong links between history and the content of the textbook. This pattern is found in books that treat advanced topics such as calculus
Anecdotic references (figures, snippets, etc.)
- Use of a few (usually short) original documents for introducing or developing a topic
- Hints for further deepening outside the textbook
- Focus on links with the development of mathematics, of science and culture.

A different cultural weight of the presence of history in the textbooks may be observed in the preceding patterns. The first pattern obliges teachers to embed their teaching in history, while in the other cases history may be used occasionally, sometimes with little integration with the goals of mathematics teaching. Thomaidis and Tzanakis (2010) have studied the official textbooks for the teaching of mathematics in the Greek high school (7th-9th grades). They found a lot of historical material, following the guidelines of the new curriculum. However, the efficacy of using them is questionable, because of serious historical errors, obscurities, or omissions.

The problem of accuracy and significance of the historical notes in the textbooks is linked to the appropriate use of history in the classroom, since, as observed in (Heiede, 1996) it may introduce myths and wrong information. The problem of the presence of history in mathematics textbook is an old one. Gebhardt (1912) wrote a extensive survey of 157 pages about the place of history in mathematics teaching in Germany. About manuals used in school his findings show that historical notes were infrequent.

The survey on the historical content in textbooks published in various countries presented by Boyé et al. (2011) points out that the presence of history in the textbooks does not guarantee that history is integrated in mathematics teaching; only when there are official regulations which encompass the introduction of history of mathematics it is not possible to avoid it in mathematics education (for example, this is the case of Denmark).

7. Conclusions and Perspectives

Summarizing what has been discussed in this chapter on the role of history of mathematics in mathematics education we may draw some general conclusions. On the one hand, if one considers mathematics teacher education, there is no doubt that to be able to put mathematics in its historical context endows teachers with a rich cultural view of the subject they teach. Both mathematical and pedagogical knowledge of teachers are enhanced since through history mathematical topics may be revisited. For the richness of situations it provides, history of mathematics is a tool in the mathematics teacher’s arsenal that may be used in suitable occasions. On the other hand, if one considers students the conclusions are more articulated. Beside the many supporters of a positive inclination to the use of history, there are researchers who express doubts. From an epistemological point of view Fried (2001) expresses his perplexities by arguing that history of mathematics and mathematics education are disciplines, each with its own aims and its own conception of the subject. Then this author deems that it is difficult to decide whether those aims are shared and those conceptions have commonalities. He claims that the need to teach students modern mathematics or mathematics as it is used in modern scientific or technological contexts
forces history of mathematics to serve aims not only foreign to its own, but even antithetical to them. The present chapter reports on examples that show that history of mathematics and mathematics education can coexist provided that the use of history is carefully planned in view of specific educational goals. The basic caveat is that not all authors, not all periods are suitable at a given situation. From a more didactical point of view Siu (2006, pp. 268-269), who plays the role of devil’s advocate, lists the following unfavorable factors presented in the form of exclamations or questions that mathematics teachers could utter when asked if they use history:

1. “I have no time for it in class!”
2. “This is not mathematics!”
3. “How can you set question on it in a test?”
4. “It can’t improve the student’s grade!”
5. “Students don’t like it!”
6. “Students regard it as history and they hate history class!”
7. “Students regard it just as boring as the subject mathematics itself!”
8. “Students do not have enough general knowledge on culture to appreciate it!”
9. “Progress in mathematics is to make difficult problems routine, so why bother to look back?”
10. “There is a lack of resource material on it!”
11. “There is a lack of teacher training in it!”
12. “I am not a professional historian of mathematics. How can I be sure of the accuracy of the exposition?”
13. “What really happened can be rather tortuous. Telling it as it was can confuse rather than to enlighten!”
14. “Does it really help to read original texts, which is a very difficult task?”
15. “Is it liable to breed cultural chauvinism and parochial nationalism?”
16. “Is there any empirical evidence that students learn better when history of mathematics is made use of in the classroom?”

Myself I have heard these objections when discussing the introduction of history with teachers. For sure the shortage of time is really a widespread problem, as evidenced by the fact that many interesting experiments that are described in literature concern the introduction of history into mathematics teaching as an optional activity outside the
regular school time, see (Testa, 1996) for one. Answering Siu’s questions make us reflect on the fact that history of mathematics is a tool not suitable to all scenarios, but that in the convenient scenario it is a powerful aid for teaching. In the following I recall some issues considered in the previous sections in order to cope with Siu’s objections. The paper (Demattè, 2007) adds further matter to my arguments.

The questions may be grouped under these main points: integration (1, 2, 3, 4), cultural understanding (5, 6, 7, 8), looking for meaning (9, 13, 14, 15), and teacher training (10, 11, 12).

- **Integration** refers to the fact that history has not to be added as an additional subject, but it has to be embedded in the teaching as it was shown in the examples presented in this paper. This means to fix educational goals and to choose moments of history and the way to deal with these moments in function of these goals. In this perspective history is a different way of doing mathematics and the problems of time, tests, and grades are overcome.

- **Cultural understanding** refers to a way of looking at mathematics as a vivid matter embedded in the socio-cultural process. History not only adds motivation, illustrations, anecdotes, but also enlightens the problematic situations behind the concepts and the theories taught in school as well as the relationship with other school disciplines and with real life. Then history introduces an epistemological dimension in the teaching and learning of mathematics. Students who do not like history (not only of mathematics, but history in general) have to acquire a "sense of history" in order to be able to interpret the world. As explicitly advocated by (Demattè, 2006), who speaks about the "strong role" that history may have in teaching, and Jankvist (2009), the introduction of history in mathematics teaching may be seen as a goal.

- **Looking for meaning** has to be one of the aims of mathematics teaching and learning. It is teachers’ endeavor to find the right tool to pursue this aim. The theoretical considerations and the examples presented in the previous sections should suggest the potentiality of history of mathematics in fostering the construction of meaning and in providing insights.

- **Teacher education** gains from the point of view of cultural understanding and of fostering pedagogical reflection. The short outline in Section 6 should hint how history of mathematics may be an alternative and efficient way of revisiting mathematical concepts and processes as well as beliefs and conceptions.

I put aside the question (16) which, indeed, reflects a main concern of researchers in mathematics education. A part the few cases in which national curricula encompass history of mathematics as a compulsory subject, experimentations of the use of history require a convenient scenario. This scenario includes: a teacher with an adequate preparation on history and with enthusiasm for trying new ways of teaching, as well as the support from the context. All that makes it difficult to establish clear empirical evidence because the particular nature of the pedagogical tool discussed in this chapter allows collecting qualitative rather than quantitative data. I have mainly anecdotal evidence through talking with many school teachers and through my personal experience as teacher trainer. However, I agree with Siu (2006, p. 276) on the fact that
"if education is really a learner-teacher-dependent endeavour, then anecdotal accounts can be as useful as, or even more than, large-scale statistical data." Of course, as advocated by Liu (2003), it is a task of the community of HPM to make as clear as possible the role of history in mathematics education.

All that said, I met a number of mathematics teachers happy for integrating history (in some of the ways presented in this chapter) in their teaching. This encourages going on in the studies on this integration. It may be discouraging to note that some doubts put forward in the pioneer attempt developed by Barwell (1913) are again mentioned in the recent attempts, but the fact that in the present days teachers are still attracted by the challenge of introducing history says something about its value. At least, one may share also today Barwell’s feeling expressed by the sentence (p. 72) "Does not even a rock appeal more to our imagination when we realise that it has a story? The subject [mathematics] is humanised; it takes a place in the pageant of our race’s history."

Appendix 1. Activities and Publications on the Use of History in Mathematics Education

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Related Chapters

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Glossary

CERME : CERME is the conference organized every two years by the European Society for Research in Mathematics Education (ERME), see http://ermeweb.free.fr/. Since 2010 ERME is an ICMI affiliate organization.

ESU : ESU is the acronym of the European Summer University (on the History and Epistemology in Mathematics Education). This activity began in 1993, following the tradition of the summer universities organized by the French Mathematics Education community, in the early 1980's. From 2010 onwards ESU takes place organized every four years. It is addressed to both researchers and teachers interested in exploring the main aspects of the presence of history and epistemology in mathematics education.

HPM : HPM is the shortened way of indicating the International Study Group on the Relations between History and Pedagogy of Mathematics affiliated to the International Commission on
Mathematical Instruction (ICMI) since 1976. By combining the history of mathematics with the teaching and learning of mathematics, HPM is the link between the past and the future of mathematics. Therefore, the group aims at stressing the conception of mathematics as a living science, a science with a long history, a vivid present and an as yet unforeseen future. Among members of the group are researchers in mathematics education, mathematicians, historians of mathematics, teachers of mathematics and curriculum developers. The history of the first 25 years of the group HPM is outlined in the section “The Affiliated Study Groups” in the website on the history of ICMI (http://www.icmihistory.unito.it/). General information about the activities of HPM and the Newsletter are in the website of HPM (http://www.clab.edc.uoc.gr/HPM/)

ICME

: ICME is the acronym of the “International Congress on Mathematical Education” held under the auspices of ICMI every four years in different sites of the world. Launched in 1969 at the initiative of ICMI President Hans Freudenthal (1905-1990), the ICMEs have been held since then in leap years. The activities carried out during this conference encompass plenary talks, plenary activities, regular lectures, topic study groups, discussion groups, workshops and sharing groups, posters, survey teams, national presentations, affiliated organizations, assemblies and presentation of ICMI activities.

ICMI

: The acronym ICMI stands for “International Commission on Mathematical Instruction”. The mission and vision of this Commission are illustrated by the following presentation in the ICMI website “Founded in 1908 to foster efforts to improve worldwide the quality of mathematics teaching and learning, the International Commission on Mathematical Instruction fulfils its mission through international programmes of activities and publications that promote reflection, collaboration, exchange and dissemination of ideas and information on all aspects of the theory and practice of contemporary mathematical education”. ICMI’s official organ is the journal L’Enseignement Mathématique, moreover the Commission publishes the ICMI Bulletin. For further information see http://www.mathunion.org/icmi.

ICMI STUDY

: Since 1984 ICMI launches every year an ICMI Study, that is a study on key topics in mathematics education, to provide an overview of studies and results in these topics. This initiative is organized as follows: the Commission launches the ICMI Study through a Discussion Document published in the journal L’Enseignement Mathématique, in the ICMI Bulletin,
and in other national and international journals; researchers submit their contributions on the theme of the Study; on the basis of the contributions received the Program Committee delivers the invitations to the ICMI Study meeting; at the end, a book (the ICMI Study volume) is published to disseminate the results. In the present chapter we refer to the ICMI Study entitled “The role of the history of mathematics in the teaching and learning of mathematics”, which was held at CIRM (Centre International de Rencontres Mathématiques) in Luminy (France), 20-25 April, 1998, see the Discussion Document (Fauvel & van Maanen, 1997) and the ICMI Study volume (Fauvel & van Maanen, 2000).

INTERNATIONAL CONGRESS OF MATHEMATICIANS: The mathematicians organize every four years the International Congress of Mathematicians (ICM) held in different sites of the world. The first one took place in 1897 in Zurich, the second in Paris in 1900. Now the organizing body is IMU (International Mathematical Union), see http://www.mathunion.org/.

IREM: The acronym IREM (Institut de Recherche sur l’Enseignement des Mathématiques) refers to the Institutes of research on the teaching of mathematics present in many French regions. The first French IREM s were created in October 1968 in Paris, Lyon and Strasbourg. Most major French universities now include an IREM. They are independent of each other, but closely collaborate inside a national network. The purpose of an IREM is to bring together teachers from primary schools, from secondary schools (general, technological or professional) and from universities to do research in common on the teaching of mathematics. The IREM s work out and circulate educational aids (such as articles, booklets, handbooks, journals, software, multi-media documents, etc.) that are based on this research and provides continuing education sessions for mathematics teachers.

Ontogeny (or ontogenesis): Ontogeny is the origin and the development of an organism, e.g. the history of an organism’s lifespan. The concept is extended so that ontogeny is defined as the history of structural changes in a unity (a cell, an organism, or a society of organisms), without the loss of the organization which allows that unity to exist.

Phylogeny (also phylogenetics or phylogenesis): Is the study of evolutionary relation among groups of organisms (e.g. species, populations).

Recapitulation: The law of recapitulation, also called the biogenetic law or embryological parallelism, is often stated as “ontogeny recapitulates phylogeny”. This law is a disproved hypothesis
formulated by Ernst Haeckel (1834-1919) for biological evolution, assuming that in developing from embryo to adult, animals go through stages resembling or representing successive stages in the evolution of their ancestors. With different formulations, such law has been extended to study several fields, such as biology, the origin of language, cognition and mental activities, anthropology, education theory and developmental psychology. In mathematics education the transposition of the recapitulation law assumes that in their intellectual development students naturally traverse more or less the same stages as mankind once did. This supposed parallelism was sometimes assumed as a support of the claim that history of mathematics can be effectively used for teaching mathematics and that history provides hints or even guidelines for organizing the curriculum.

Bibliography


Arcavi, A., Bruckheimer, M., & Ben-Zvi, R. (1982). Maybe a mathematics teacher can profit from the study of the history of mathematics. For the Learning of Mathematics, 3(1), 30-37. [In this article the authors show how a course based on history may be useful for making teachers reflect on mathematical concepts]


Barbin, E. (coordinator), Furinghetti, F., Lawrence, S., & Smestad, B. (2011). The role of the history and epistemology of mathematics in pre-service teachers training. In E. Barbin, M. Kronfellner, & C. Tzanakis (Eds.), History and Epistemology in Mathematics Education. Proceedings of the Sixth European Summer University ESU 6 (pp. 27-46). Vienna: Verlag Holzhausen GmbH / Holzhausen Publishing Ltd. [This contribution illustrates the state of art of teacher training about history of mathematics in four countries and presents some possible models]

Barwell, M. E. (1913). The advisability of including some instruction in the school course on the history of mathematics. The Mathematical Gazette, 7, 72-79. [This article reports on the introduction of some historical notes in the author’s classroom]


Boyé, A., Dematté, A., Lakoma, E., Tzanakis, C. (2011). The history of mathematics in school textbooks. In E. Barbin, M. Kronfellner, C. Tzanakis (Eds.), History and Epistemology in Mathematics Education. Proceedings of the Sixth European Summer University ESU 6 (pp. 153-163). Vienna: Verlag Holzhausen GmbH / Holzhausen Publishing Ltd. [This contribution illustrates if and how history is present in the textbooks of authors’ countries]


Burn, B. (1993). Individual development and historical development: a study of calculus. *International Journal of Mathematical Education in Science and Technology, 24*, 429-433. [This paper contrasts the way of presenting calculus to students between the ages of 16 and 18 in England with the rigor of undergraduate courses in analysis]

Burn, B. (1999). Integration, a genetic approach. *Nordisk Matematikk Didaktikk, 7*(1), 7–27. [This paper describes a teaching sequence inspired by the genetic method]

Burn, B. (2005). The vice: Some historical inspired and proof-generated steps to limits of sequences. *Educational Studies in Mathematics, 60*, 269–295. [This paper proposes a genetic development of the concept of limit of a sequence leading to a definition, through a succession of proofs. The major ideas on which it is based depend on Euclid, Archimedes, Fermat, Wallis and Newton]


Clark, K. M. (2011). History of mathematics: illuminating understanding of school mathematics concepts for prospective mathematics teachers. *Educational Studies in Mathematics, DOI 10.1007/s10649-011-9361-y*. [This paper presents the results of a research about how prospective mathematics teachers know the topics that they will teach and how that teaching might include an historical component]


Demattè, A., & Furinghetti, F. (2011). History, figures, and narratives in mathematics teaching. *MAA series*. In V. Katz, & C. Tzanakis (Eds.), *Recent developments on introducing a historical dimension in mathematics education* (pp. 103-112). Washington, DC: Mathematical Association of America. [This paper presents an experiment in which figures taken from historical mathematics textbooks are used in the mathematics classroom]

Dias, I. C. (2008). From the original text of Pedro Nunes to the mathematics classroom activities. In E. Barbin, N. Stehlíková, & C. Tzanakis (Eds.), *Proceedings of the 5th European Summer University ESU 5*(pp. 259-260). Plzeň: Vydavatelský servis. [This contribution describes the use of Portuguese discoveries in mathematics teaching]


Farmaki, V., & Paschos, T. (2007). Employing genetic ‘moments’ in the history of mathematics in classroom activities. *Educational Studies in Mathematics, 66*, 83–106. [This paper utilizes Oresme’s genetic ideas on motion and geometry to develop mathematical models that can be employed for the solution of problems relating to linear motion]


Fraser, B. J., & Koop, A. J. (1978). Teachers’ opinion about some teaching materials involving history of mathematics. *International Journal of Mathematical Education in Science and Technology, 9*, 147-151. [This paper investigates how teachers were ready to use in their classroom some historical materials presented to them in a training course]


Freudenthal, H. (1981). Should a mathematics teacher know something about the history of mathematics?. *For the Learning of Mathematics, 2*(1), 30-33. [This article considers the role that history of mathematics may have in teachers’ knowledge]

Fried, M. N. (2001). Can mathematics education and history of mathematics coexist?. *Science & Education, 10*, 391-408. [This paper attempts to solve the problem of combining two disciplines such as history and education which are developed in different tracks]

Furinghetti, F. (1997). History of mathematics, mathematics education, school practice: case studies linking different domains. *For the learning of mathematics, 17*(1), 55-61. [This paper identifies the main goals of the introduction of history in mathematics teaching and provides a few examples]


Mathematics, 66, 131-143. [This paper discusses how history to mathematics is used in teacher training]

Furinghetti, F. (to appear). Part IV, Chapter XXIII. History of international cooperation in mathematics education. In A. Karp, & G. Schubring (Eds.), Handbook on history of mathematics education. New York, NY: Springer. [This chapter illustrates the evolution of mathematics education from local and national initiatives to the establishment of international bodies of cooperation]


Furinghetti, F. & Somaglia, A. (1997). Storia della matematica in classe. L’educazione matematica, s. 5, 2, 26-46. [This paper analyzes various examples of the use of history in the classroom with particular reference to the method of analysis and synthesis]

Gargani, G. (1996). Un percorso artistico. Lettera Pristem, 22, 47. [This paper is a short report on an experiment of using history for acting on students’ image of mathematics]


Gravemeijer, K., & Doorman, M. (1999). Context problems in realistic mathematics education: a calculus course as an example, Educational Studies in Mathematics, 39, 111-129. [This paper describes an experiment in which concept of calculus are introduced through using history]

Gulikers, I. (2002-03). The seventeenth-century surveyor in class. BSHM Newsletter, 47, 56-63. [This article describes how old techniques and instruments were used to introduce similarity]

Gulikers, I., & Blom, K. (2001). ‘A historical angle’, a survey of recent literature on the use and value of history in geometrical education. Educational Studies in Mathematics, 47, 223-258. [This article presents a survey of the literature reporting on the use and value of the history in geometrical education]


Heppel, G. (1893). Nineteenth general report of the Association for the Improvement of Geometrical
Teaching. Bedford: W. J. Robinson, 19-33. See also Heppel, G. (1893). The use of history in teaching mathematics. Nature, 48, 16-18. [This paper discusses how history of mathematics may be efficiently used in the classroom and provides some examples. The cultural value of history of mathematics is stressed]


Hsieh, F.-J. (2000). Teachers’ teaching beliefs and their knowledge about the history of negative numbers. In W.-S. Horng & F.-L. Lin (Eds.), Proceedings of the HPM 2000 Conference History in mathematics education: Challenges for a new millennium. A satellite meeting of ICME-9 (Vol. 1, pp. 88-97) Taipei, Taiwan: Department of Mathematics, National Taiwan Normal University. [This paper studies the changes in teachers’ view of negative number after learning their history]

Hunger Parshall, K. (1988). The art of algebra from Al-Khwarizmi to Viète: a study in the natural selection of ideas. History of science, 26(72), 129-164. [This paper outlines the evolution of algebraic thinking]


Klein, F. (1939). Elementary mathematics from an advanced standpoint. Part I: Arithmetic, Algebra, Analysis. Part II: Geometry. Translated by E. R. Hedrick and C. A. Noble. New York: Dover Publications. [In this work, based on the notes of the courses he delivered in 1907-1908, the famous mathematician outlines the mathematical content to be developed for teacher education]

Kool, M. (1992). Dust clouds from the 16th century. The Mathematical Gazette, 76(475), 90-96. [This paper reports on an experiment in which Dutch arithmetic texts of 16th century were used to motivate students with learning difficulties]


Liu, Po-Hung (2003). Do teachers need to incorporate the history of mathematics in their teaching?. Mathematics Teacher, 96, 370-377. [This paper attempts to answer the question "Why should the history of mathematics have a place in school mathematics?" through theoretical arguments and empirical evidence]

Lorey, W. (1926). Gustav Eneström. Isis, 8, 313-320. [The obituary of the founder of the pioneer journal on the history of mathematics Bibliotheca Mathematica outlines some characters of the research in this field during the period of publication of the journal]


Percival, I. (2001). An artefactual approach to ancient arithmetic. For the Learning of Mathematics, 21(3), 16-21. [This article contains the account of an activity about arithmetic of ancients in the classroom]


Radford, L. (1997). On psychology, historical epistemology, and teaching of mathematics: towards a socio-cultural history of mathematics. *For the learning of mathematics*, 17(1), 26-33. [This article is a wide exposition of the author’s ideas about history and mathematics education]


Radford, L., & Guérette, G. (2000). Second degree equations in the classroom: A Babylonian approach. In V. Katz (Ed.), *Using history to teach mathematics. An international perspective* (pp. 69–75). Washington, DC: Mathematical Association of America. [This paper describes a teaching sequence whose purpose is to lead the students to reinvent the formula that solve the general quadratic equation]

Ransom, P. (1995). Navigation and surveying: teaching geometry through the use of old instruments. In F. Lalande, F. Jaboeuf, & Y. Nouazé, (Eds.) (1995). *Actes de la première Université d’Été Européenne. Histoire et Épistémologie dans l’Éducation Mathématique* (pp. 227-239). Montpellier, France: IREM de Montpellier, Université Montpellier II. [The author reports on a workshop where easily made instruments such as sundials and the cross-staff were used for teaching trigonometry and geometry]


Schubring, G. (2006). Ontogeny and phylogeny - Categories for cognitive development. In F. Furinghetti, S. Kaijser, & C. Tzanakis (Eds.), *Proceedings HPM 2004 & ESU 4 – Revised edition* (pp. 329-339). Iraklion, Greece: University of Crete. [In this paper classical and recent texts on the relation of phylogeny and ontogeny are presented and discussed, with special emphasis on categories relevant for cognitive development]


Siu, M.-K. (2006). No, I don’t use history of mathematics in my class: Why?. In F. Furinghetti, S. Kaijser, & C. Tzanakis (Eds.), *Proceedings HPM 2004 & ESU 4 – Revised edition* (pp. 268-277). Iraklion, Greece: University of Crete. [This paper discusses some possible objections made by teachers to the use of history of mathematics in their classes]

Skemp, R. (1969). *The psychology of learning mathematics*. Harmondsworth: Penguin. [This is a famous treatise on learning mathematics written by a psychologist]

Smestad, B. (2011). Teachers’ conceptions of history of mathematics. In V. Katz, & C. Tzanakis (Eds.),
**Recent developments on introducing a historical dimension in mathematics education** (pp. 233-242). Washington, DC: Mathematical Association of America. [This paper describes an interview study on Norwegian teachers’ conceptions of the history of mathematics]


Stander, D. (1989). The use of the history of mathematics in teaching. In P. Ernest (Ed.), *Mathematics teaching. The state of the art* (pp. 241-246). New York, NY: The Falmer Press. [This paper analyses two short experiments on the use of history with students and with prospective teachers. The conclusion is that in both cases little improvement in attitude towards mathematics was observed]

Swetz, F. J. (1995). To know and to teach: mathematical pedagogy from a historical context. *Educational Studies in Mathematics*, 29, 73-88. [In this article pedagogical issues are linked to ancient documents from Babylonia and China]


Tymoczko, T. (1994). Humanistic and utilitarian aspects of mathematics. In D. F. Robitaille, D. H. Wheeler, & C. Kieran (Eds.), *Selected lectures from the 7th International Congress on Mathematical Education* (pp. 327-339). Québec: Les Presses de l’Université Laval. [This paper discusses the aspects of the nature of mathematics]


Tzanakis, C., & Thomaides, Y. (2011). Complementary routes to integrate history in mathematics education: In search of an appropriate theoretical framework. In E. Barbin, M. Kronfellner, C. Tzanakis (Eds.), *History and Epistemology in Mathematics Education. Proceedings of the Sixth European Summer University ESU 6* (pp. 127-137). Vienna: Verlag Holzhausen GmbH / Holzhausen Publishing Ltd. [The authors attempt a classification of the use of history in mathematics teaching]

Van Amerom, B. (2003). Focusing on informal strategies when linking arithmetic to early algebra. *Educational Studies in Mathematics*, 54, 63-75. [This paper presents a project for treating the transition from arithmetic to algebra using history]


a humanistic orientation]

**Biographical Sketch**

**Fulvia Furinghetti** is full professor of Mathematics Education in the Department of Mathematics of the University of Genoa (Italy). Her research concerns mathematics education and history of mathematics education. In the first years of her career she carried out research in projective-differential geometry. She developed projects on the use of history of mathematics in teaching, the history of mathematics education, beliefs, the public image of mathematics, proof and problem solving, algebra, technology in mathematics education, teacher professional development, teachers’ and student’s beliefs. She has organized the Symposia celebrating the Centenary of *L’Enseignement Mathématique* (Geneva, 2000) and of ICMI (Rome, 2008) and edited the proceedings of both Symposia. She is the author (with Livia Giacardi) of the website on the first hundred years of ICMI http://www.icmihistory.unito.it/. In 2000-2004 she chaired HPM, the International Study Group on the relations between History and Pedagogy of Mathematics affiliated to ICMI (International Commission on Mathematical Instruction).

**To cite this chapter**


5.1. Stream A: History for Promoting Mathematics

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Appendix 1. Activities and Publications on the Use of History in Mathematics Education

In addition to what has been presented in the text of the chapter, this appendix supplies some further information. The following list of activities and publications is far from exhaustive; in particular, it does not include the many websites concerning the topic in question. Nevertheless it is hoped that it offers useful hints for personal search of materials about the use of history in mathematics education.

A) In each ICME conference there are official activities dedicated to history in mathematics education (Working Groups, Topic Study Groups, HPM special sessions). Moreover Satellite meetings are held in a place close to the venue of ICME. The following proceedings of the Satellite meetings have been published:


B) European Summer Universities (ESUs) are held in different European countries; they are particularly addressed at teachers. Some are organized in conjunction with Satellite meetings. The following proceedings have been published:


2. ESU-2: see Satellite meeting of 1996.


Republic: Vydavatelsky servis, Prague. (One volume).


**C) Books containing selected papers presented in Satellite meetings, European Summer Universities, or other meeting related to history and pedagogy of mathematics:**


**D) Some special issues of journals on the use of history in mathematics teaching:**

Journals of education publish papers on the use of history in mathematics education, and, in particular, some special issues entirely devoted to this topic. Some of these special issues are:

- *Mathematics Teacher*, 93(8), 2000.

**E) Activities in conferences**
During conferences on general themes of mathematics education there are talks, working groups and other activities related to history in mathematics education. In particular, working groups on this subject were organized during CERME 6 and CERME 7 (CERME stands for Congress of the European Society for Research in Mathematics Education.) Sometimes in the conferences on history of mathematics and on mathematics there are talks which deal with the use of history in mathematics education.

Regular activities and conferences on history in mathematics education are carried out in France by the IREM network and in the UK by the British Society for the History of Mathematics. The French IREMs publish proceedings of their meetings, reprints of historical texts, and teaching materials. Specific conferences on history in mathematics education are organized around the world. Among them, it is interesting the Brazilian initiative launched in 2002 called Colóquio de História e Tecnologia no Ensino de Matemática (HTEM). This conference on history and technology in mathematics teaching has the aim of creating a space of discussion about the impact of research on history and about the role of technology in mathematics teaching.

The section of the website on the history of ICMI (http://www.icmihistory.unito.it/) dedicated to the Study Group HPM contains the history of the first 25 years of HPM. This history reports on the main conferences organized in the period in question. The international Newsletter of HPM and the regional Newsletters of HPM published in the US and in Taiwan (HPM Tongxun) provide information about the various initiatives at national and international level.